Appendices
for
“Slow-burn” Spillover and “Fast and Furious” Contagion: A Study of International Stock Markets

Lei Wu

Qingbin Meng

Kuan Xu*

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*Corresponding authors: mengqingbin@rbs.org.cn and kuan.xu@dal.ca. Lei Wu (School of Economics and Management, Beihang University, No.37 Xueyuan Road, Beijing 100191, l.wu@buaa.edu.cn) thanks the National Natural Science Foundation of China for support (Grant No. 71303016). Qingbin Meng (School of Business, Renmin University of China, No. 59 Haidian Road, Beijing, China 100872) thanks the National Natural Science Foundation of China, Ministry of Education’s Humanities and Social Sciences Projects and Beijing City Board of Education’s Young Talent Plan for support (Grant No. 31702156 and 10YJC790196). Kuan Xu (Department of Economics, Dalhousie University, Halifax, Nova Scotia, Canada B3H 4R2) thanks School of Business, Renmin University of China for support for its summer visitor program. The authors wish to thank the Editors of Quantitative Finance and anonymous referees for constructive comments and suggestions. The authors also wish to thank Wachindra Bandara, Lawrence Booth, Iraj Fooladi, Talan Iscan, Robert Jung, Robert Maderitsch, Leonard MacLean, Luis Filipe Martins, Maria Pacurar, John Rumsey, Ke Tang, Yonggan Zhao, Jun Zhou, and other participants at the School of Business at Dalhousie University, Hanqing Advanced Institute of Economics and Finance at Renmin University, the Midwest Finance Association 2012 Meetings, the Eastern Finance Association 2013 Meetings, and the 11th INIFINITI Conference on International Finance for their constructive comments. The remaining errors are ours.

Appendix A

In order to establish the theoretical basis for our analysis, we attempt to propose a theoretical setup for two stock markets. This setup gives us some insight as to how we can study contagion when two market portfolio indices are cointegrated. Then we implement a simple simulation to reinforce this insight.

In our theoretical setup, we attempt to characterize the cointegration relation between two stock markets 1 and 2, whose prices at time t, p_{1t} and p_{2t}, can be modelled, respectively, as:

$$d\ln p_{1t} = \alpha_1 d\ln p_{2t} + \beta_1 \left(a + \ln p_{1t} - b\ln p_{2t}\right) dt + dB_{1t} \tag{A-1}$$

and

$$d\ln p_{2t} = \alpha_2 d\ln p_{1t} + \beta_2 \left(c + \ln p_{2t} - d\ln p_{1t}\right) dt + dB_{2t}, \tag{A-2}$$

where α_1, α_2, β_1, β_2, a, b, c, and d are constant parameters and B_{1t} and B_{2t} are composite Wiener processes given by

$$B_{1t} = \sigma_{11}W_{1t} + \sigma_{12}W_{2t} \tag{A-3}$$

and

$$B_{2t} = \sigma_{21}W_{1t} + \sigma_{22}W_{2t}. \tag{A-4}$$

Let $\ln p_t = [\ln p_{1t}, \ln p_{2t}]^T$. Equations (A-1) and (A-2) can be presented in the matrix form as the multivariate:

$$A d\ln p_t = \mu dt + C\ln p_t dt + \Sigma dW_t, \tag{A-5}$$

where $A = \begin{bmatrix} 1 & -\alpha_1 \\ -\alpha_2 & 1 \end{bmatrix}$, $dW_t = [dW_{1t}, dW_{2t}]^T$, $\mu = [\beta_1 a, \beta_2 c]^T$, $C = \begin{bmatrix} \beta_1 & -\beta_1 b \\ -\beta_2 d & \beta_2 \end{bmatrix}$, and $\Sigma = \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{bmatrix}$. From equation (A-5), we have

$$d\ln p_t = A^{-1}\mu dt + A^{-1}C\ln p_t dt + A^{-1}\Sigma dW_t, \tag{A-6}$$

which can be further simplified into

$$d\ln p_t = (\tilde{\mu} - \tilde{C}\ln p_t)dt + \tilde{\Sigma} dW_t. \tag{A-7}$$

This expression implies that for the two stock market prices to be cointegrated their prices must be governed by the same stochastic trend (e.g., the growth of the global capital markets) and be perturbed by shocks from same sources (e.g., common/correlated shocks to the global capital markets). This is the basis for us to study contagion.

To see the above point clearer, we derive $\ln p_t$. Given that the matrix exponential, $e^{-\tilde{C}t}$, exists, from equation (A-7) we obtain the following expression:

$$e^{-\tilde{C}t} d\ln p_t + e^{-\tilde{C}t}\tilde{C}\ln p_t dt = e^{-\tilde{C}t}\tilde{\mu}dt + e^{-\tilde{C}t}\tilde{\Sigma}dW_t. \tag{A-8}$$

Rewriting the left-hand side of equation (A-8), we have

$$de^{-\tilde{C}t}\ln p_t = e^{-\tilde{C}t}\tilde{\mu}dt + e^{-\tilde{C}t}\tilde{\Sigma}dW_t. \tag{A-9}$$

Now we integrate equation (A-9) from 0 to t resulting:

$$e^{-\tilde{C}t}\ln p_t - \ln p_0 = \int_0^t e^{-\tilde{C}s}\tilde{\mu}ds + \int_0^t e^{-\tilde{C}s}\tilde{\Sigma}dW_s, \tag{A-10}$$

where x_0 is the initial value of $\{x_t, t \geq 0\}$. We simplify the first term on the right-hand side of equation (A-10) to get

$$e^{-\tilde{C}t}\ln p_t - \ln p_0 = -\frac{\tilde{\mu}}{\tilde{C}}(e^{-\tilde{C}t} - \mathbf{1}) + \int_0^t e^{-\tilde{C}s}\tilde{\Sigma}dW_s, \tag{A-11}$$

where $\mathbf{1}$ is a square matrix with elements being unity. Further simplifying equation (A-11) yields

$$\ln p_t = e^{\tilde{C}t}\ln p_0 - \frac{\tilde{\mu}}{\tilde{C}}(\mathbf{1} - e^{\tilde{C}t}) + \int_0^t e^{-\tilde{C}(s-t)}\tilde{\Sigma}dW_s. \tag{A-12}$$

This expression further implies that for the two stock market prices to be cointegrated their prices must be governed by the same stochastic trend and be perturbed by the common/correlated shocks. As we note, the two market prices are not cointegrated either because they are not governed by the same stochastic trend, or are perturbed by uncommon/uncorrelated shocks, or both. In this case, we could not study contagion which is defined as the transmission of some shocks beyond their long-term interdependence.

In the following simulation, we propose a simple framework in which two stock markets, market 1 and 2, are affected by common factors (without the deterministic trend) but, sometimes, are affected by market-specific factors of their own. p_{it} is the market portfolio index (in log) for market i in period t. As can be seen later, this discussion can be easily extended to the case of n markets.

We assume that the market portfolio prices for these two markets are determined by two k-factor models.[1] That is, the market portfolio price for market i ($i = 1, 2$) has the following data generate process: for $t = 1, 2, \ldots, T$,

$$p_{it} = \sum_{j=1}^{k} \mathtt{b}_{ij} f_{jt} + v_{it}, \tag{A-13}$$

where f_{jt}, $j = 1, 2, \ldots, k$, are the k common factors and v_{it} is the innovation unique to market i ($i = 1, 2$). Here, $p_{it} \sim I(1)$, $f_{jt} \sim I(1)$, and $v_{it} \sim I(0)$. The first difference of the k-factor model for the stock market portfolio price leads to the k-factor model of the stock market portfolio return. That is, let $r_{it} = \Delta p_{it} = p_{i,t} - p_{i,t-1}$ is the market portfolio return for stock market i in period t. Then the k-factor model of the stock market portfolio price is $r_{it} = \sum_{j=1}^{k} \mathtt{b}_{ij} \Delta f_{jt} + \Delta v_{it}$.

The k-factor model in equation (A-13) assumes that, other than market specific risk factor v_{it}, there is no factor that is unique to a specific market and that the common factors jointly affect the two stock markets. If there exists such a market specific factor x_{1t} that systematically influences market 1 but not market 2,[2] the price equation for market 1 must be changed to

$$p_{1t} = \sum_{j=1}^{k} \mathtt{b}_{1j} f_{jt} + \mathtt{b}_{1,k+1} x_{1t} + v_{1t}. \tag{A-14}$$

The price equation for market 2 remains to be

$$p_{2t} = \sum_{j=1}^{k} \mathtt{b}_{2j} f_{jt} + v_{2t}. \tag{A-15}$$

The data generating processes for p_{1t} and p_{2t} are quite different when x_{1t} is $I(1)$ but is also a near $I(2)$. This reflects that stock market 1 experiences a growth pattern that differs from that of stock market 2.[3]

If $\mathtt{b}_{1,k+1} = 0$ in equation (A-14), the two market portfolio prices (p_{1t} and p_{2t}) share the

[1] In our simulation exercise, for simplicity, we assume that two market portfolio prices are regulated by their k-factor models.

[2] For easy of communication, we let market 1 to be exposed to x_{1t} in addition to the common factors f_{jt}, where $j = 1, 2, \ldots, k$ and $t = 1, 2, \ldots, T$. Please note that this is a simplification. Logically, this is equivalent to let market i to be exposed to x_{it} but market 1 is exposed to $x'_{1t} = x_{1t} - x_{2t}$.

[3] Here, a $I(1)$ but near $I(2)$ process x_{1t} can be used to describe this growth pattern. Such a process can be generated from $z_t \sim I(0)$ using $(1 - L)(1 - \rho L)x_{1t} = z_t$, where $|\rho| + \epsilon = 1$ and ϵ is a very small number. This implies $x_{1t} = (1 + \rho)x_{1,t-1} - \rho x_{1,t-2} + z_t$.

common factors and are cointegrated. If $\mathtt{b}_{1,k+1} \neq 0$, although these prices share the common factors, they may not be cointegrated because of the presence of the factor that is unique to market 1, x_{1t}. Our simulation results show that when x_{1t} has more influence on p_{1t} than f_{1t} and f_{2t} do, the cointegration between p_{1t} and p_{2t} may be weakened substantially. In this case, we cannot establish the long-run equilibrium between the two stock markets and hence we cannot evaluate contagion which is viewed as a departure from this long-run equilibrium. We will report our simulation exercise and results at the latter part of the appendix.

In practice, because we cannot observe the common and market specific factors, we can only rely on the identification of a cointegration relation between the two market portfolio prices to establish the long-run equilibrium. The cointegration relation can take one of the following forms:

$$p_{1t} = \alpha_1 + \beta_1 p_{2t} + e_{1t} \tag{A-16}$$

and

$$p_{2t} = \alpha_2 + \beta_2 p_{1t} + e_{2t}, \tag{A-17}$$

where α_k and β_k are cointegrating parameters and e_{kt} is the error term in period t in cointegration relation k for $k = 1, 2$. These error terms reflect deviations from these cointegration relations.[4]

Now we study e_{kt} for $k = 1, 2$ in period t. Substituting equation (A-13) into equations (A-16) and (A-17), we obtain

$$\sum_{j=1}^{k} \mathtt{b}_{1j} f_{jt} + v_{1t} = \alpha_1 + \beta_1 \left(\sum_{j=1}^{k} \mathtt{b}_{2j} f_{jt} + v_{2t} \right) + e_{1t}, \tag{A-18}$$

and

$$\sum_{j=1}^{k} \mathtt{b}_{2j} f_{jt} + v_{2t} = \alpha_2 + \beta_2 \left(\sum_{j=1}^{k} \mathtt{b}_{1j} f_{jt} + v_{1t} \right) + e_{2t}. \tag{A-19}$$

Now we express e_{kt}, $k = 1, 2$, as functions of v_{1t} and v_{2t}:

$$e_{1t} = \delta_{1t} + (v_{1t} - \beta_1 v_{2t}) \tag{A-20}$$

and

$$e_{2t} = \delta_{2t} + (v_{2t} - \beta_2 v_{1t}), \tag{A-21}$$

[4]One may note that $\alpha_2 = -\frac{\alpha_1}{\beta_1}$, $\beta_2 = \frac{1}{\beta_1}$, and $e_{2t} = -\frac{e_{1t}}{\beta_1}$.

where

$$\delta_{1t} = -\alpha_1 + \sum_{j=1}^{k} (\mathtt{b}_{1j} - \beta_1 \mathtt{b}_{2j}) f_{jt} \tag{A-22}$$

and

$$\delta_{2t} = -\alpha_2 + \sum_{j=1}^{k} (\mathtt{b}_{2j} - \beta_2 \mathtt{b}_{1j}) f_{jt}. \tag{A-23}$$

Equations (A-20) and (A-21) represent the deviations from the cointegration relations given in equations (A-16) and (A-17). In addition, the error terms of cointegration regressions, e_{1t} and e_{2t}, are $I(0)$. Because, for $i = 1, 2$, $E(v_{it}) = 0$ and $E(e_{it}) = 0$, then $E(\delta_{it}) = 0$. That is, the error terms of the k-factor models are expected to be zero and the residuals of the cointegration regressions are expected to be zero. These facts also imply that the two stock market portfolio prices are cointegrated if no other market specific factors to disturb specific markets.

Although the k-factor models give us some traction on the underlying data generating processes for the market portfolio prices, we cannot observe the common and market specific factors. However, we can use the factor models to make sense the cointegration relations shown by equations (A-16) and (A-17) and identify α_k, β_k, and e_{kt}, $k = 1, 2$. In addition, we also want to make sense of how the residuals from the cointegration relations, e_{1t} and e_{2t}, are related to the unobservable market specific factors embedded in the underlying factor models, v_{1t} and v_{2t}.

Please note that the general setup for n stock markets can be explained by the case of two stock markets which are affected by a set of common and market specific factors. Following our theoretical discussion, we can stack the error terms of cointegration regressions for two stock markets, e_{1t} and e_{2t}, into

$$\mathbf{e}_t = \begin{bmatrix} e_{1t} \\ e_{2t} \end{bmatrix}. \tag{A-24}$$

Now we can identify how $\mathbf{e}_t$ is related to v_{it} $(i = 1, 2)$, the orthogonal structural innovations to markets 1 and 2, v_{1t} and v_{2t}, respectively. Now we stack them into

$$\mathbf{v}_t = \begin{bmatrix} v_{1t} \\ v_{2t} \end{bmatrix}. \tag{A-25}$$

We can relate $\mathbf{v}_t$ to $\mathbf{e}_t$ using the following structure:

$$\mathbf{e}_t = \mathbf{A}^{-1}\mathbf{B}\mathbf{v}_t. \tag{A-26}$$

where

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \tag{A-27}$$

and

$$\mathbf{B} = \begin{bmatrix} b_{11} & 0 \\ 0 & b_{22} \end{bmatrix}. \tag{A-28}$$

Let $\gamma = \frac{1}{|\mathbf{A}|}$. Because

$$\mathbf{A}^{-1} = \begin{bmatrix} \gamma a_{22} & -\gamma a_{12} \\ -\gamma a_{21} & \gamma a_{11} \end{bmatrix}, \tag{A-29}$$

$$\begin{aligned} \begin{bmatrix} e_{1t} \\ e_{2t} \end{bmatrix} &= \begin{bmatrix} \gamma a_{22} & -\gamma a_{12} \\ -\gamma a_{21} & \gamma a_{11} \end{bmatrix} \begin{bmatrix} b_{11} & 0 \\ 0 & b_{22} \end{bmatrix} \begin{bmatrix} v_{1t} \\ v_{2t} \end{bmatrix} \\ &= \begin{bmatrix} \gamma a_{22} b_{11} v_{1t} - \gamma a_{12} b_{22} v_{2t} \\ -\gamma a_{21} b_{11} v_{1t} + \gamma a_{11} b_{22} v_{2t} \end{bmatrix}. \end{aligned} \tag{A-30}$$

Expanding the above expression as two separate equations, we obtain

$$e_{1t} = \gamma a_{22} b_{11} v_{1t} - \gamma a_{12} b_{22} v_{2t} \tag{A-31}$$

and

$$e_{2t} = \gamma a_{11} b_{22} v_{2t} - \gamma a_{21} b_{11} v_{1t} \tag{A-32}$$

We see that equations (A-31) and (A-32) share the similar structures of equations (A-20) and (A-21) if $\delta_i = 0$, $i = 1, 2$. In addition to $\delta_i = 0$, $i = 1, 2$, if we further impose the restrictions $\gamma a_{22} b_{11} = \gamma a_{11} b_{22} = 1$, $\gamma a_{12} b_{22} = \beta_1$, and $\gamma a_{21} b_{11} = \beta_2$, then equations (A-31) and (A-32) share the identical structures of equations (A-20) and (A-21).

A simulation exercise can be used to illustrate the validity of using the cointegration analysis to identify equilibrium relations among stock market prices when they are influenced by a set of common factors. When some stock market prices are driven more by their market specific factors, the identification of such equilibrium relations could be difficult.

To specify the parameters in the simulation exercise for the data generating processes given in equations (A-14) and (A-15), we assume that the number of the factors is $k = 2$. The sample size is $T = 1000$. The bi-factor models have the following parameters: $\mathtt{b}_{11} = 0.2$, $\mathtt{b}_{12} = 0.2$, $\mathtt{b}_{13} = 0.4$, $\mathtt{b}_{21} = 0.1$, and $\mathtt{b}_{22} = 0.3$. The factor 1, f_{1t}, is generated by $(1-L)f_{1t} = w_{1t} \sim N(2,4)$. The factor 2, f_{2t}, is generated by $(1-L)f_{2t} = w_{2t} \sim N(1,1)$. The error terms of the bi-factor models are $u_{1t} \sim N(0,1)$ and $u_{2t} \sim N(0,1)$, respectively, and they are statistically independent. In addition, we let x_{1t} be a $I(1)$ but near $I(2)$ process. Such a process can be generated from $z_t \sim I(0)$ using $(1-L)(1-\rho L)x_{1t} = z_t$, where $|\rho| + \epsilon = 1$ and ϵ is a very small number. This implies $x_{1t} = (1+\rho)x_{1,t-1} - \rho x_{1,t-2} + z_t$. In our simulation exercise, we let $\rho = 0.97$.

Figures A1 and A2 show the changes in two common factors, f_{1t} and f_{2t}. These two factors jointly influence two stock market portfolio returns and, therefore, their prices p_{1t} and p_{2t}. Figure A3 shows x_{1t}, which is $I(1)$ but near $I(2)$ with $\rho = 0.97$. This factor enters the data generating process of p_{1t} when $\mathtt{b}_{13} = 0.4$ (see Figure A4). When $\mathtt{b}_{13} = 0$, this factor does not enter the data generating process of p_{1t} (see Figure A5). In this simulation exercise, we do not allow p_{2t} to be affected by another market specific factor beyond the two common factors f_{1t} and f_{2t}. As can be seen in Figures A4 and A5, the return for stock market 1 portfolio can be affected by the factor that is specific to market 1.

Now we examine the plausible cointegration relation between the two stock market portfolio prices. As shown in Figure A7, the two prices appear to be not cointegrated when the stock market 1 portfolio price is influenced by x_{1t}. As shown in Figure A8, the two prices appear to be cointegrated when the stock market 1 portfolio price is not influenced by x_{1t}.

The examination of the residuals of this cointegration regression based on the graphical analysis (see Figure A9) and the cointegration test further confirms that the two prices under this condition are not cointegrated. If we eliminate the impact of x_{1t} on the stock market 1 portfolio price, we can find a cointegration relation between the two prices based on the graphical analysis (see Figure A10) and the cointegration test. Of course, the latter is completely expected as the two prices are influenced jointly by the two common factors.

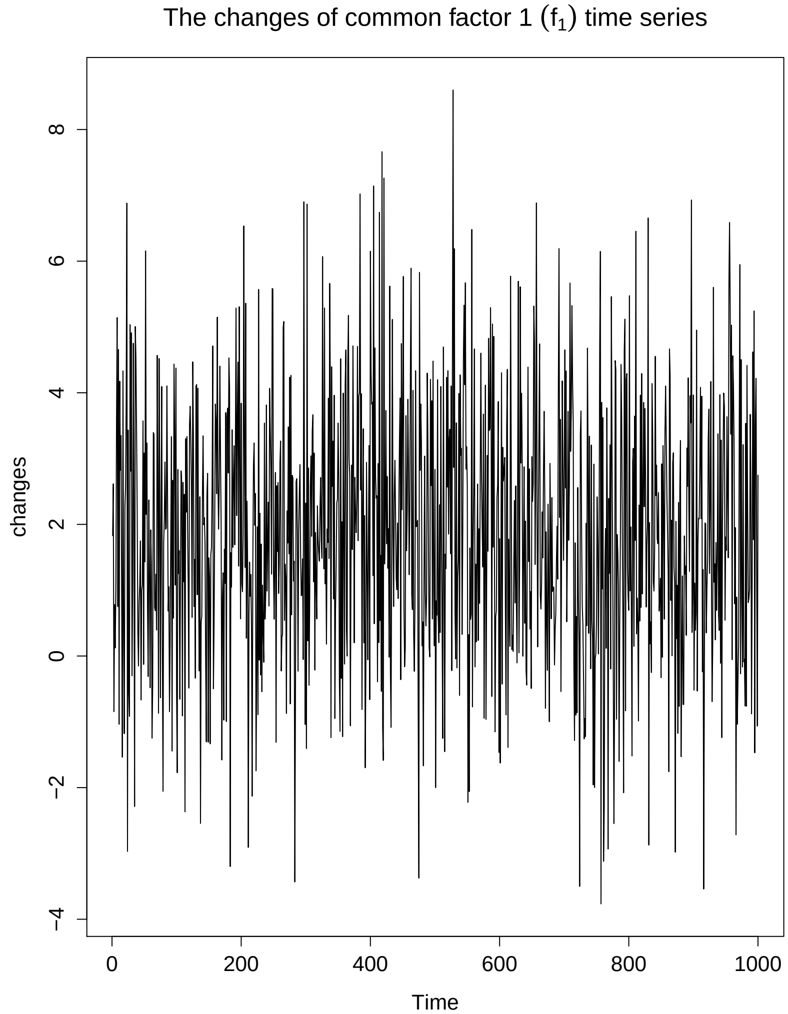

Figure A1: Changes of common factor f_1

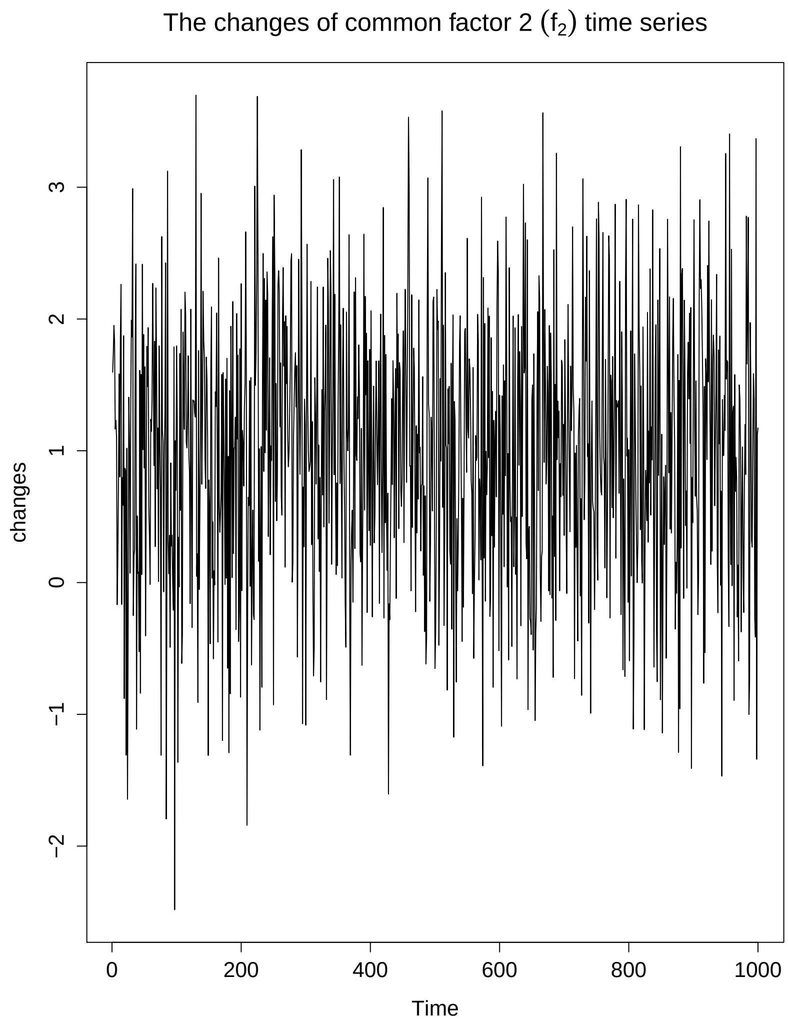

Figure A2: Changes of common factor f_2

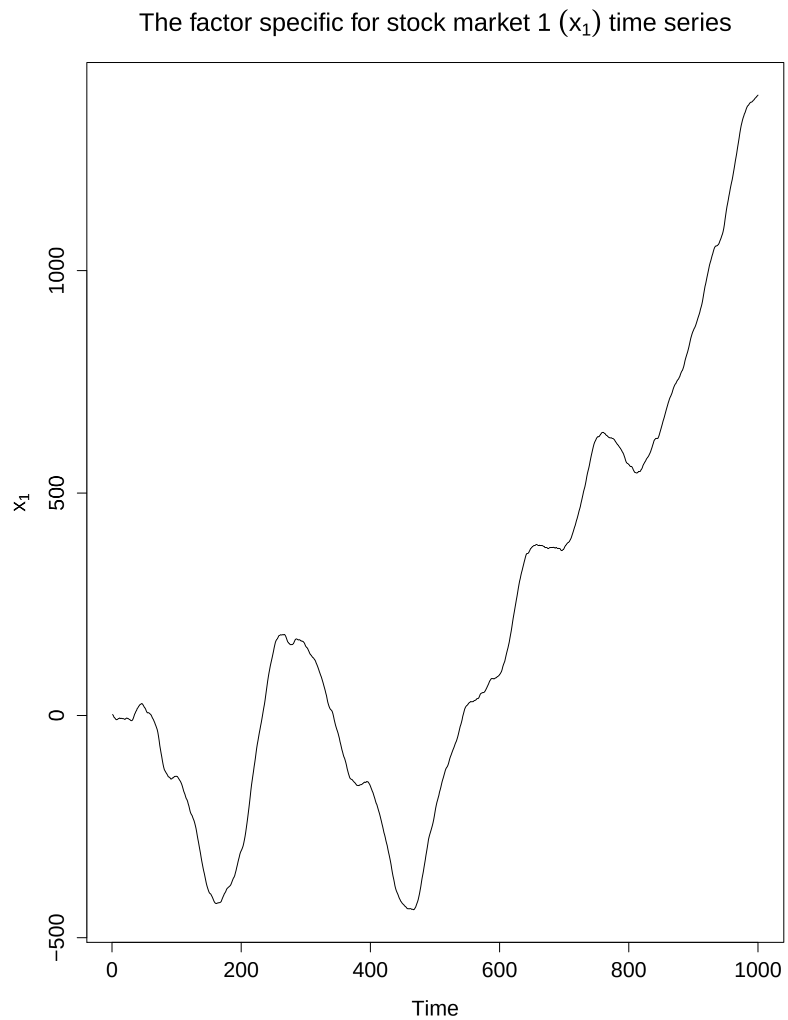

Figure A3: Factor specific for market 1, x_1

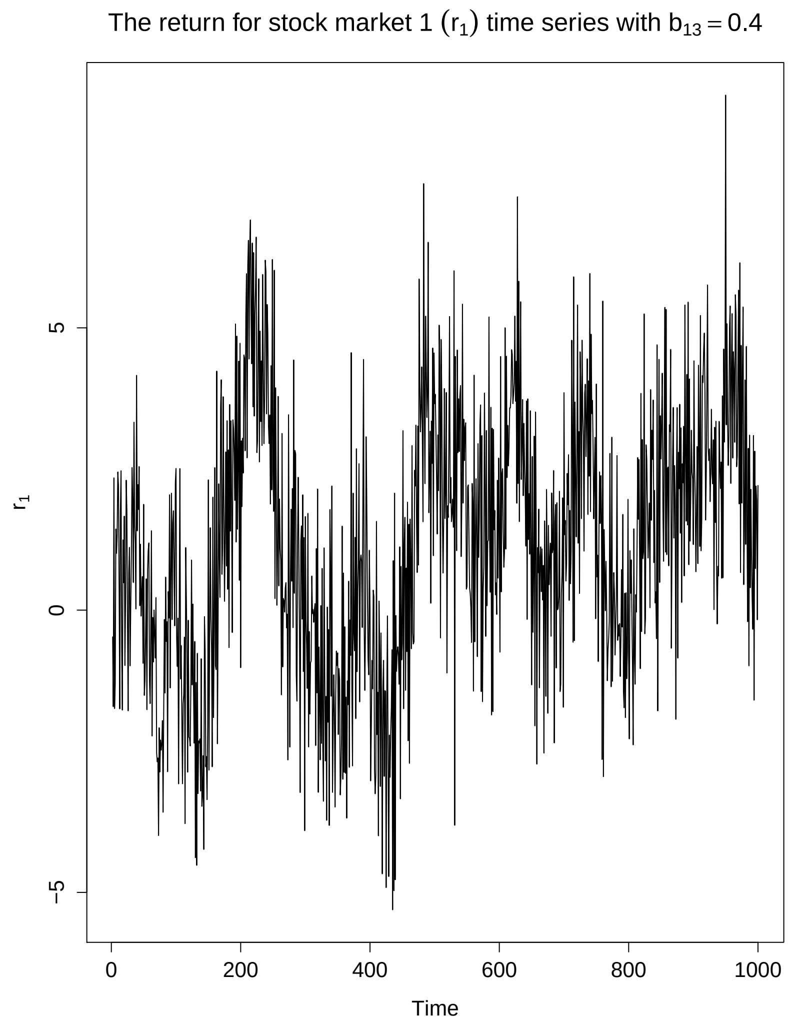

Figure A4: Market 1 portfolio return influenced by x_1

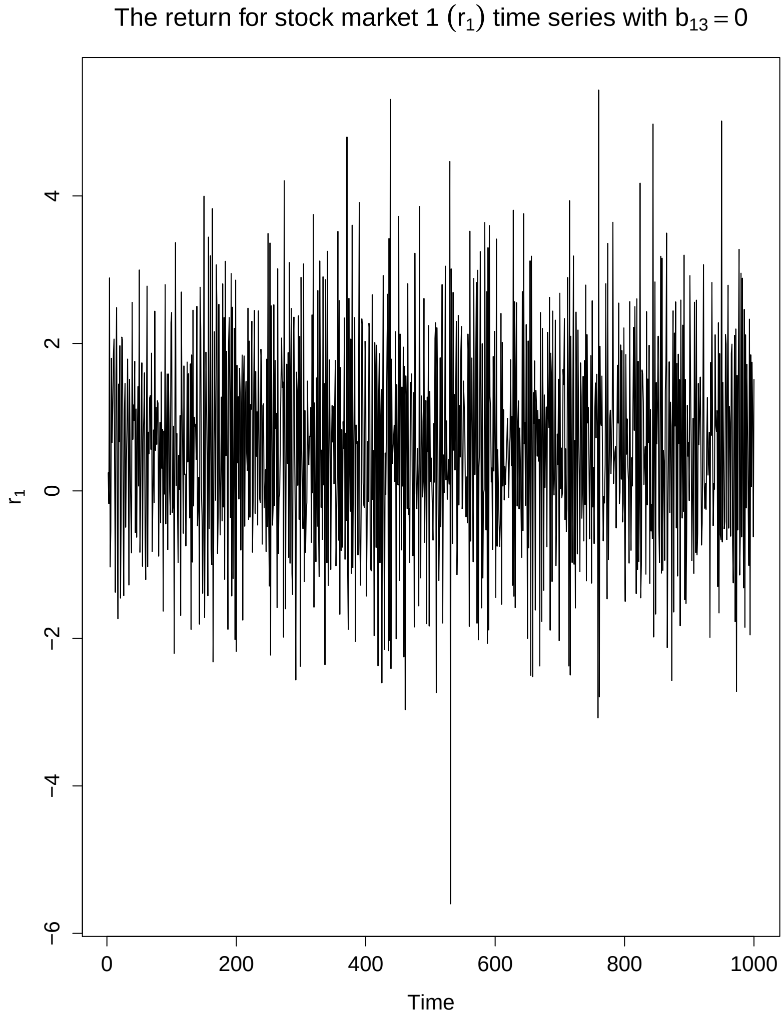

Figure A5: Market 1 portfolio return not influenced by x_1

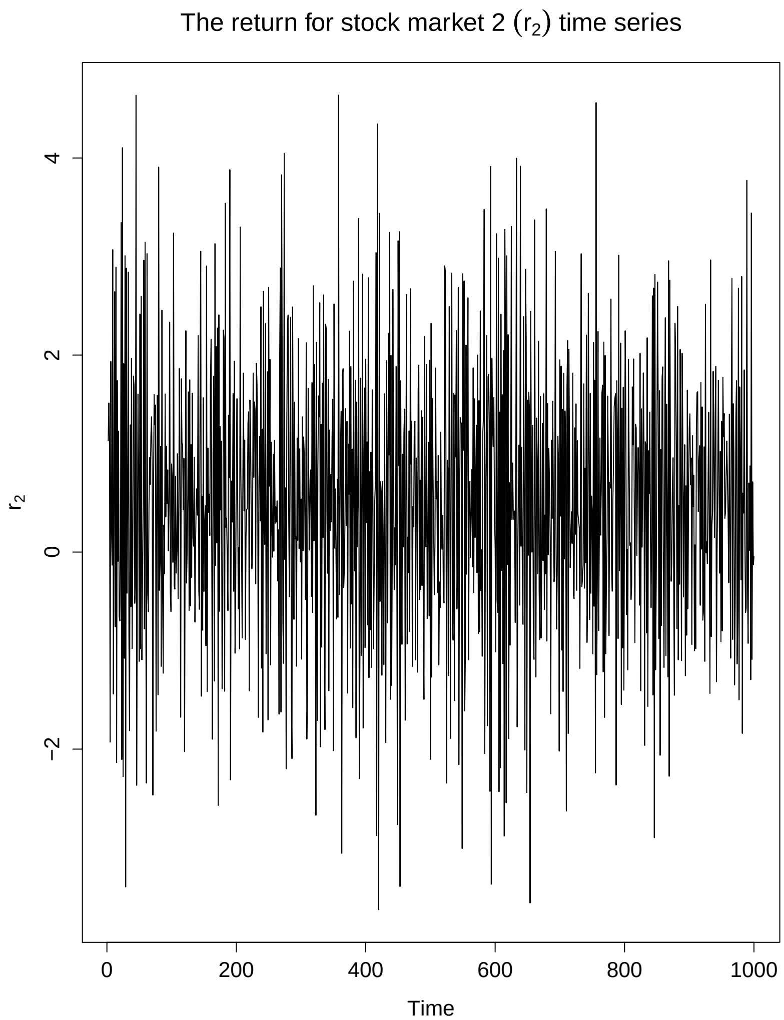

Figure A6: Market 2 portfolio return

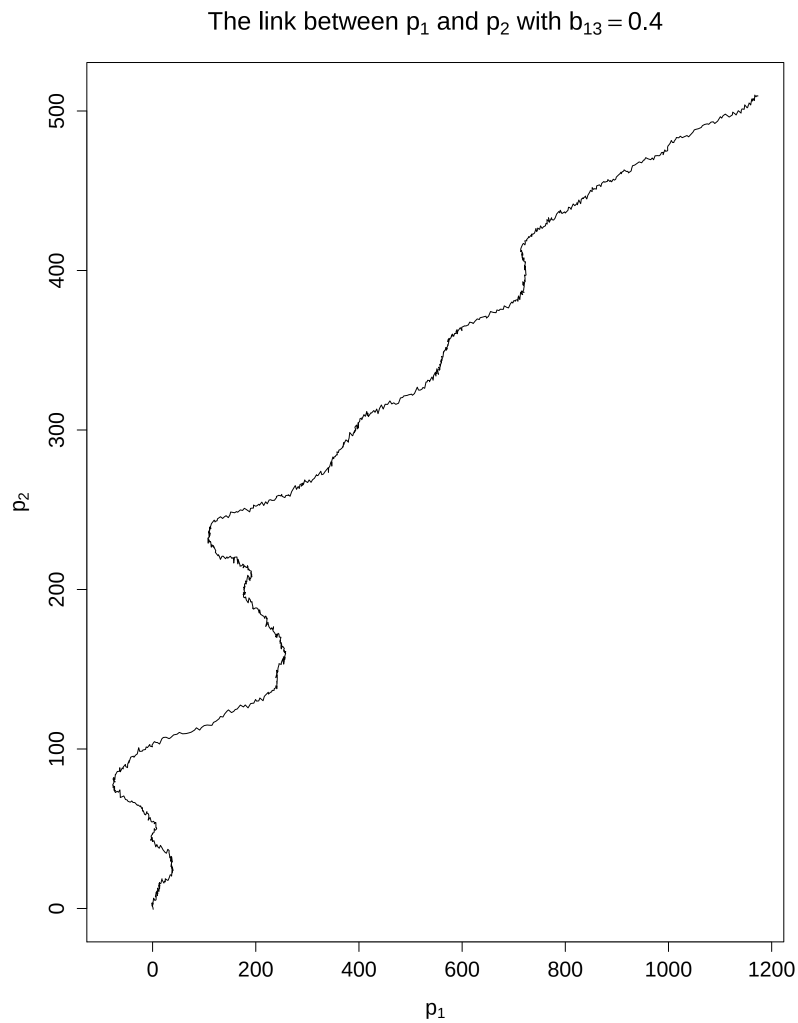

Figure A7: The link between two market portfolio prices with market 1 influenced by x_1

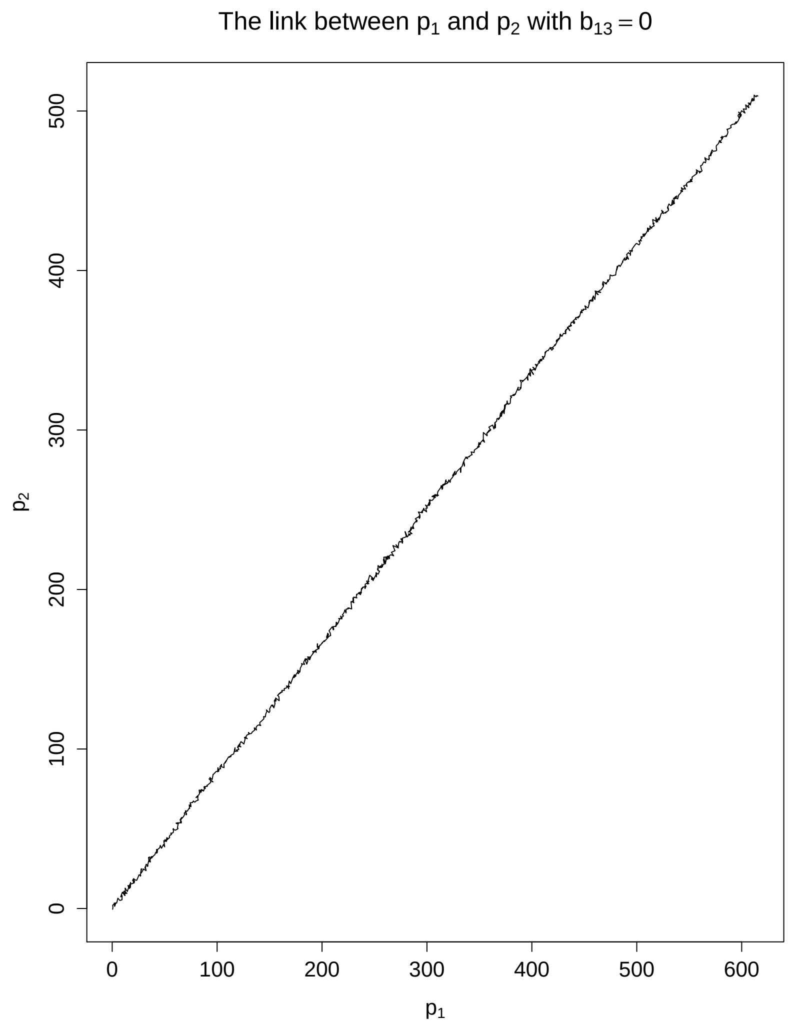

Figure A8: The link between two market portfolio prices with market 1 not influenced by x_1

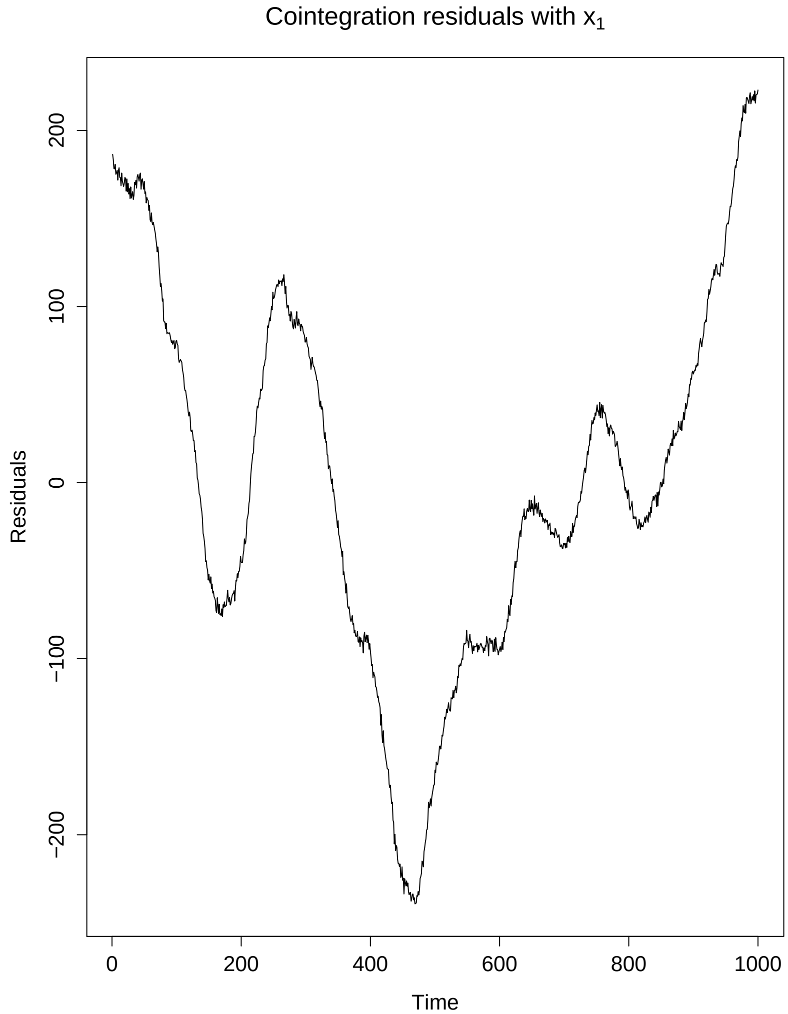

Figure A9: Non-stationary residuals in the cointegration regression with market 1 influenced by x_1

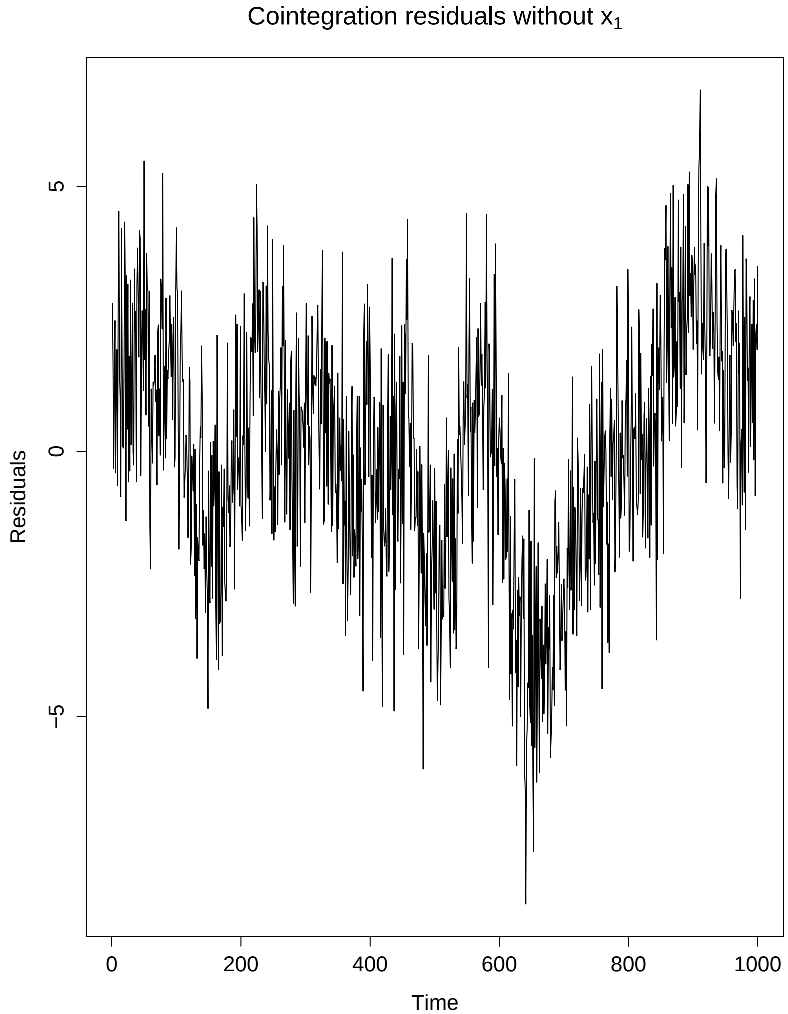

Figure A10: Stationary residuals in the cointegration regression with market 1 not influenced by x_1

Appendix B

Without loss of generality, we assume that stock market portfolio returns follow the data generating process of $\Delta\mathbf{p}_t = \mathbf{\Pi}\mathbf{p}_{t-1} + \mathbf{\Gamma}\Delta\mathbf{p}_{t-1} + \mathbf{e}_t$. This data generating process permits dynamics and comovements within and across stock markets. The VECM has a VAR representation

$$\mathbf{A}(L)\mathbf{p}_t = \mathbf{e}_t \tag{B-1}$$

where $\mathbf{A}(L) = \mathbf{I}_n - \mathbf{A}_1 L - \mathbf{A}_2 L^2$ with $\mathbf{A}_1 = \mathbf{\Pi} + \mathbf{I}_n + \mathbf{\Gamma} = \alpha\beta' + \mathbf{I}_n + \mathbf{\Gamma}$ and $\mathbf{A}_2 = -\mathbf{\Gamma}$. Here α is an $n \times r$ matrix and β an $n \times r$ matrix capturing the r cointegration relations among n elements in $\mathbf{p}_t$.

Gonzalo and Granger (1995) define $\Delta\mathbf{P}_t$ and $\Delta\mathbf{T}_t$ as the innovations associated with the permanent (P) and transitory (T) components of $\Delta\mathbf{p}_t$, respectively. Their P-T decomposition is as follows:

$$\Delta\mathbf{p}_t = \Delta\mathbf{P}_t + \Delta\mathbf{T}_t = \theta_1\Delta\mathbf{f}_t + \theta_2\Delta\mathbf{z}_t, \tag{B-2}$$

where $\theta_1 = \beta_\perp(\alpha'_\perp\beta_\perp)^{-1}$ and $\theta_2 = \alpha(\beta'\alpha)^{-1}$ so that θ_1 is an $n \times (n-r)$ matrix and θ_2 is an $n \times r$ matrix. $\mathbf{f}_t = \alpha'_\perp\mathbf{p}_t$ and $\mathbf{z}_t = \beta'\mathbf{p}_t$.

Let $\mathbf{G} = \begin{bmatrix} \alpha'_\perp \\ \beta' \end{bmatrix}$, then $\mathbf{G}\mathbf{p}_t = \begin{bmatrix} \mathbf{f}_t \\ \mathbf{z}_t \end{bmatrix}$. Thus, we have

$$\mathbf{G}\mathbf{A}(L)\mathbf{G}^{-1}\begin{bmatrix} (1-L)\mathbf{I}_{n-r} & 0 \\ 0 & \mathbf{I}_r \end{bmatrix}^{-1}\begin{bmatrix} \Delta\mathbf{f}_t \\ \mathbf{z}_t \end{bmatrix} = \mathbf{G}\mathbf{A}(L)\mathbf{G}^{-1}\begin{bmatrix} \mathbf{f}_t \\ \mathbf{z}_t \end{bmatrix} = \mathbf{G}\mathbf{A}(L)\mathbf{p}_t = \mathbf{G}\mathbf{e}_t. \tag{B-3}$$

Equation (B-3) is the AR representation of $\begin{bmatrix} \Delta\mathbf{f}_t \\ \mathbf{z}_t \end{bmatrix}$. To write it in an extensive form, define the first $n-r$ columns of $\mathbf{G}^{-1}$ as $\mathbf{G}^{-1}_{n-r}$ and the last r columns of $\mathbf{G}^{-1}$ as $\mathbf{G}^{-1}_r$. Then we have

$$\begin{bmatrix} \mathbf{I}_{n-r} - (\alpha'_\perp\mathbf{\Gamma}\mathbf{G}^{-1}_{n-r})L & (-\alpha'_\perp\mathbf{\Gamma}\mathbf{G}^{-1}_r)L(1-L) \\ (-\beta'\mathbf{\Gamma}\mathbf{G}^{-1}_{n-r})L & \mathbf{I}_r - (\beta'\alpha + \mathbf{I}_r + \beta'\mathbf{\Gamma}\mathbf{G}^{-1}_r)L + (\beta'\mathbf{\Gamma}\mathbf{G}^{-1}_r)L^2 \end{bmatrix}\begin{bmatrix} \Delta\mathbf{f}_t \\ \mathbf{z}_t \end{bmatrix} = \mathbf{G}\mathbf{e}_t. \tag{B-4}$$

We can write equation (B-4) compactly as

$$\begin{bmatrix} \mathbf{F}_{11}(L) & \mathbf{F}_{12}(L) \\ \mathbf{F}_{21}(L) & \mathbf{F}_{22}(L) \end{bmatrix}\begin{bmatrix} \Delta\mathbf{f}_t \\ \mathbf{z}_t \end{bmatrix} = \mathbf{G}\mathbf{e}_t \tag{B-5}$$

$\mathbf{F}_{ij}(L)$ can be derived according to equation (B-3). For example, $\mathbf{F}_{11}(L)$ is an $(n-r)\times(n-r)$

matrix which is from "(the first $n-r$ rows of $\mathbf{G}$) $\times\mathbf{A}(L)\times$ (the first $n-r$ columns of $\mathbf{G}^{-1}$)/$(1-L)$". We can obtain $\mathbf{F}_{12}(L)$, $\mathbf{F}_{21}(L)$ and $\mathbf{F}_{22}(L)$ similarly. Let $L=0,1$, we have

$$\begin{aligned}
\mathbf{F}_{11}(0) &= \mathbf{I}_{n-r}, \quad \mathbf{F}_{11}(1)=\mathbf{I}_{n-r}-\alpha'_{\perp}\mathbf{\Gamma}\mathbf{G}_{n-r}^{-1}.\\
\mathbf{F}_{12}(0) &= \mathbf{0}, \qquad \mathbf{F}_{12}(1)=\mathbf{0},\\
\mathbf{F}_{21}(0) &= \mathbf{0}, \qquad \mathbf{F}_{21}(1)=-\beta'\mathbf{\Gamma}\mathbf{G}_{n-r}^{-1},\\
\mathbf{F}_{22}(0) &= \mathbf{I}_{r}, \qquad \mathbf{F}_{22}(1)=-\beta'\alpha.
\end{aligned} \tag{B-6}$$

Let $\mathbf{u}_t^P=\alpha'_{\perp}\mathbf{e}_t$ and $\mathbf{u}_t^T=\beta'\mathbf{e}_t$. We can write equation (B-5) as

$$\begin{bmatrix} \mathbf{F}_{11}(L) & \mathbf{F}_{12}(L) \\ \mathbf{F}_{21}(L) & \mathbf{F}_{22}(L) \end{bmatrix} \begin{bmatrix} \Delta\mathbf{f}_t \\ \mathbf{z}_t \end{bmatrix} = \begin{bmatrix} \mathbf{u}_t^P \\ \mathbf{u}_t^T \end{bmatrix}. \tag{B-7}$$

Inverting equation (B-7) we obtain

$$\begin{bmatrix} \Delta\mathbf{f}_t \\ \mathbf{z}_t \end{bmatrix} = \begin{bmatrix} \mathbf{F}^{11}(L) & \mathbf{F}^{12}(L) \\ \mathbf{F}^{21}(L) & \mathbf{F}^{22}(L) \end{bmatrix} \begin{bmatrix} \mathbf{u}_t^P \\ \mathbf{u}_t^T \end{bmatrix}, \tag{B-8}$$

where $\begin{bmatrix} \mathbf{F}^{11}(L) & \mathbf{F}^{12}(L) \\ \mathbf{F}^{21}(L) & \mathbf{F}^{22}(L) \end{bmatrix} = \begin{bmatrix} \mathbf{F}_{11}(L) & \mathbf{F}_{12}(L) \\ \mathbf{F}_{21}(L) & \mathbf{F}_{22}(L) \end{bmatrix}^{-1}$. We assume that $\mathbf{F}^{ij}(L)$'s exist and can be determined by inverting the partitioned matrix. Therefore, we have

$$\begin{aligned}
\mathbf{F}^{11}(L) &= (\mathbf{F}_{11}(L)-\mathbf{F}_{12}(L)\mathbf{F}_{22}(L)^{-1}\mathbf{F}_{21}(L))^{-1}, \qquad \text{(B-9)}\\
\mathbf{F}^{12}(L) &= -(\mathbf{F}_{11}(L)-\mathbf{F}_{12}(L)\mathbf{F}_{22}(L)^{-1}\mathbf{F}_{21}(L))^{-1}\mathbf{F}_{12}(L)\mathbf{F}_{22}(L)^{-1},\\
\mathbf{F}^{21}(L) &= -\mathbf{F}_{22}(L)^{-1}\mathbf{F}_{21}(L)(\mathbf{F}_{11}(L)-\mathbf{F}_{12}(L)\mathbf{F}_{22}(L)^{-1}\mathbf{F}_{21}(L))^{-1},\\
\mathbf{F}^{22}(L) &= \mathbf{F}_{22}(L)^{-1}+\mathbf{F}_{22}(L)^{-1}\mathbf{F}_{21}(L)(\mathbf{F}_{11}(L)-\mathbf{F}_{12}(L)\mathbf{F}_{22}(L)^{-1}\mathbf{F}_{21}(L))^{-1}\mathbf{F}_{12}(L)\mathbf{F}_{22}(L)^{-1}.
\end{aligned}$$

Let $L=0,1$, we have

$$\begin{aligned}
\mathbf{F}^{11}(0) &= \mathbf{I}_{n-k}, \quad \mathbf{F}^{11}(1)=\mathbf{F}_{11}(1)^{-1},\\
\mathbf{F}^{12}(0) &= \mathbf{0}, \qquad \mathbf{F}^{12}(1)=\mathbf{0},\\
\mathbf{F}^{21}(0) &= \mathbf{0}, \qquad \mathbf{F}^{21}(1)=-\mathbf{F}_{22}(1)^{-1}\mathbf{F}_{21}(1)\mathbf{F}_{11}(1)^{-1}\\
\mathbf{F}^{22}(0) &= \mathbf{I}_{r}, \qquad \mathbf{F}^{22}(1)=\mathbf{F}_{22}(1)^{-1}.
\end{aligned} \tag{B-10}$$

Furthermore, we can express $\Delta\mathbf{P}_t$ and $\Delta\mathbf{T}_t$ equation by equation compactly as

$$\begin{aligned}
\begin{bmatrix} \Delta\mathbf{P}_t \\ \Delta\mathbf{T}_t \end{bmatrix} &= \begin{bmatrix} \theta_1 & 0 \\ 0 & \theta_2 \end{bmatrix} \begin{bmatrix} \Delta\mathbf{f}_t \\ \Delta\mathbf{z}_t \end{bmatrix} \\
&= \begin{bmatrix} \theta_1 & 0 \\ 0 & \theta_2 \end{bmatrix} \begin{bmatrix} \mathbf{I}_{n-r} & 0 \\ 0 & (1-L)\mathbf{I}_r \end{bmatrix} \begin{bmatrix} \mathbf{F}^{11}(L) & \mathbf{F}^{12}(L) \\ \mathbf{F}^{21}(L) & \mathbf{F}^{22}(L) \end{bmatrix} \begin{bmatrix} \mathbf{u}_t^P \\ \mathbf{u}_t^T \end{bmatrix} \\
&= \begin{bmatrix} \theta_1\mathbf{F}^{11}(L) & \theta_1\mathbf{F}^{12}(L) \\ \theta_2(1-L)\mathbf{F}^{21}(L) & \theta_2(1-L)\mathbf{F}^{22}(L) \end{bmatrix} \begin{bmatrix} \mathbf{u}_t^P \\ \mathbf{u}_t^T \end{bmatrix}.
\end{aligned} \tag{B-11}$$

Therefore,

$$\begin{aligned}
\Delta\mathbf{P}_t &= \theta_1\mathbf{F}^{11}(L)\mathbf{u}_t^P + \theta_1\mathbf{F}^{12}(L)\mathbf{u}_t^T, \\
\Delta\mathbf{T}_t &= \theta_2(1-L)\mathbf{F}^{21}(L)\mathbf{u}_t^P + \theta_2(1-L)\mathbf{F}^{22}(L)\mathbf{u}_t^T.
\end{aligned} \tag{B-12}$$

It is worth noting that $\mathbf{F}^{12}(1) = \mathbf{0}$ in equation (B-10), which implies that the permanent shock $\Delta\mathbf{P}_t$ still has the transitory component $\theta_1\mathbf{F}^{12}(L)\mathbf{u}_t^T$.

Substituting equations (B-12) into equation (B-2), we have

$$\begin{aligned}
\Delta\mathbf{p}_t &= \Delta\mathbf{P}_t + \Delta\mathbf{T}_t \\
&= \begin{bmatrix} \theta_1\mathbf{F}^{11}(L) + \theta_2(1-L)\mathbf{F}^{21}(L) & \theta_1\mathbf{F}^{12}(L) + \theta_2(1-L)\mathbf{F}^{22}(L) \end{bmatrix} \begin{bmatrix} \mathbf{u}_t^P \\ \mathbf{u}_t^T \end{bmatrix} \\
&= \begin{bmatrix} \mathbf{D}_1(L) & \mathbf{D}_2(L) \end{bmatrix} \begin{bmatrix} \mathbf{u}_t^P \\ \mathbf{u}_t^T \end{bmatrix}.
\end{aligned} \tag{B-13}$$

Here we define

$$\begin{aligned}
\mathbf{D}_1(L) = \theta_1\mathbf{F}^{11}(L) + \theta_2(1-L)\mathbf{F}^{21}(L). \\
\mathbf{D}_2(L) = \theta_1\mathbf{F}^{12}(L) + \theta_2(1-L)\mathbf{F}^{22}(L).
\end{aligned} \tag{B-14}$$

Because $\mathbf{F}^{12}(1) = \mathbf{0}$, $\mathbf{D}_2(1) = \mathbf{0}$, which means that $\mathbf{u}_t^T$ only has transitory effect on the level of $\mathbf{p}_t$. Hence, $\mathbf{u}_t^P$ and $\mathbf{u}_t^T$ are named the permanent and transitory shocks, respectively, by Gonzalo and Ng (2001). This P-T decomposition differs from that of Gonzalo and Granger (1995).

However, if we focus on the components of $\mathbf{D}_1(L)$, we find that the permanent shock still has transitory component. Let $L = 0, 1$. We have

$$\begin{aligned}
\mathbf{D}_1(0) &= \theta_1\mathbf{F}^{11}(0) + \theta_2\mathbf{F}^{21}(0) = \theta_1\mathbf{I}_{n-r}, \\
\mathbf{D}_2(0) &= \theta_1\mathbf{F}^{12}(0) + \theta_2\mathbf{F}^{22}(0) = \theta_2\mathbf{I}_r, \\
\mathbf{D}_1(1) &= \theta_1\mathbf{F}^{11}(1) = \theta_1\mathbf{F}_{11}(1)^{-1}, \\
\mathbf{D}_2(1) &= \theta_1\mathbf{F}^{12}(1) = \mathbf{0}.
\end{aligned} \tag{B-15}$$

Here, $\mathbf{D}_1(0) = \theta_1\mathbf{I}_{n-r}$ is the initial impact of a permanent shock, $\mathbf{u}_t^P$. $\mathbf{D}_1(1) = \theta_1\mathbf{F}_{11}(1)^{-1}$ is the long-run pricing impact. Only when $\mathbf{F}_{11}(1)^{-1} = \mathbf{I}_{n-r}$, i.e., $\mathbf{\Gamma} = \mathbf{0}$, the initial impact of $\mathbf{u}_t^P$ is equal to its long-run pricing impact.

As we can see, different P-T decomposition methods always provide different identifications for permanent and transitory shocks. In fact, all the information can be fully reflected in the level of $\mathbf{p}_t$ if given long-enough period. What we focus on, especially in contagion analysis, should be the deviation of the initial impact of innovations on stock markets from the long-run pricing impact and why this deviation exists. Therefore, we only consider the deviation of the initial impact from the long-run pricing impact and avoid using the terms "permanent shocks" and "transitory shocks," which could be ambiguous in our context.

The $n \times 1$ vector of error terms, $\mathbf{e}_t$, can be expressed in a structural relation with the $n \times 1$ vector of unobservable structural innovations $\mathbf{v}_t$: $\mathbf{A}\mathbf{e}_t = \mathbf{B}\mathbf{v}_t$, where $\mathbf{v}_t \sim (\mathbf{0}, \mathbf{I}_n)$ and $\mathbf{A}$ and $\mathbf{B}$ are $n \times n$ matrices of structural parameters. Some parameters are restricted to 0 or 1 for identification while others are to be estimated. $\mathbf{A}$ contains the contemporaneous correlation coefficients among error terms while $\mathbf{B}$ is a diagonal matrix containing the standard deviations of structural innovations. Substituting $\mathbf{e}_t = \mathbf{A}^{-1}\mathbf{B}\mathbf{v}_t$ into equation (B-13), we obtain

$$\begin{aligned}
\Delta\mathbf{p}_t &= \begin{bmatrix} \mathbf{D}_1(L) & \mathbf{D}_2(L) \end{bmatrix} \begin{bmatrix} \alpha'_\perp \\ \beta' \end{bmatrix} \mathbf{A}^{-1}\mathbf{B}\mathbf{v}_t \\
&= \mathbf{D}_1(L)\alpha'_\perp\mathbf{A}^{-1}\mathbf{B}\mathbf{v}_t + \mathbf{D}_2(L)\beta'\mathbf{A}^{-1}\mathbf{B}\mathbf{v}_t. \qquad \text{(B-16)}
\end{aligned}$$

According to equation (B-15), the initial impact of the structural innovation $\mathbf{v}_t$ on the level of $\mathbf{p}_t$ is $\mathbf{D}_1(0)\alpha'_\perp\mathbf{A}^{-1}\mathbf{B} + \mathbf{D}_2(0)\beta'\mathbf{A}^{-1}\mathbf{B} = \theta_1\mathbf{I}_{n-r}\alpha'_\perp\mathbf{A}^{-1}\mathbf{B} + \theta_2\mathbf{I}_r\beta'\mathbf{A}^{-1}\mathbf{B}$. The long-run pricing impact is $\mathbf{D}_1(1)\alpha'_\perp\mathbf{A}^{-1}\mathbf{B} = \theta_1\mathbf{F}_{11}(1)^{-1}\alpha'_\perp\mathbf{A}^{-1}\mathbf{B}$. Therefore, equation (B-16) can be further decomposed into

$$
\begin{aligned}
\Delta \mathbf{p}_t &= \begin{bmatrix} \mathbf{D}_1(L) & \mathbf{D}_2(L) \end{bmatrix} \begin{bmatrix} \alpha'_\perp \\ \beta' \end{bmatrix} \mathbf{A}^{-1}\mathbf{B}\mathbf{v}_t \\
&= \mathbf{D}_1(L)\alpha'_\perp \mathbf{A}^{-1}\mathbf{B}\mathbf{v}_t + \mathbf{D}_2(L)\beta'\mathbf{A}^{-1}\mathbf{B}\mathbf{v}_t \qquad \text{(B-17)} \\
&= \underbrace{\mathbf{D}_1(1)\alpha'_\perp \mathbf{A}^{-1}\mathbf{B}\mathbf{v}_t}_{\left(\substack{\text{Long-run Pricing Impact} \\ \text{Denoted } \mathbf{\Phi}\mathbf{v}_t}\right)} + \underbrace{\Big(\mathbf{D}_1(L)\alpha'_\perp \mathbf{A}^{-1}\mathbf{B} - \mathbf{D}_1(1)\alpha'_\perp \mathbf{A}^{-1}\mathbf{B} + \mathbf{D}_2(L)\beta'\mathbf{A}^{-1}\mathbf{B}\Big)\mathbf{v}_t}_{\left(\substack{\text{Pricing Error} \\ \text{Denoted}\mathbf{\Phi}^*(L)\mathbf{v}_t, \text{ and } \mathbf{\Phi}^*(1)\mathbf{v}_t = \mathbf{0}}\right)}
\end{aligned}
$$

where the long-run pricing impact of innovations $\mathbf{v}_t$ is measured by a matrix of scalars, $\mathbf{\Phi}$, and the pricing error induced by the innovations has a dynamic effect, $\mathbf{\Phi}^*(L)$, which satisfies $\mathbf{\Phi}^*(0) = \theta_1 I_{n-r}\alpha'_\perp \mathbf{A}^{-1}\mathbf{B} - \theta_1 \mathbf{F}_{11}(1)^{-1}\alpha'_\perp \mathbf{A}^{-1}\mathbf{B} + \theta_2 \mathbf{I}_r \beta' \mathbf{A}^{-1}\mathbf{B}$ and $\mathbf{\Phi}^*(1) = \mathbf{0}$.

Finally, when $\mathbf{\Gamma} = \mathbf{0}$ and $\beta'\mathbf{A}^{-1}\mathbf{B} = \mathbf{0}$,

$$
\begin{aligned}
\mathbf{\Phi}^*(0) &= \mathbf{D}_1(0)\alpha'_\perp \mathbf{A}^{-1}\mathbf{B} - \mathbf{D}_1(1)\alpha'_\perp \mathbf{A}^{-1}\mathbf{B} + \mathbf{D}_2(0)\beta'\mathbf{A}^{-1}\mathbf{B} \\
&= \theta_1 \mathbf{I}_{n-r}\alpha'_\perp \mathbf{A}^{-1}\mathbf{B} - \theta_1 \mathbf{I}_{n-r}\alpha'_\perp \mathbf{A}^{-1}\mathbf{B} + \mathbf{0} \\
&= \mathbf{0} \qquad \text{(B-18)}
\end{aligned}
$$

When $\mathbf{\Gamma} = \mathbf{0}$, then no long-run auto-correlations exist among the elements in $\mathbf{p}_t$. This implies high efficiency of stock markets. When $\beta'\mathbf{A}^{-1}\mathbf{B} = \mathbf{0}$, then the contemporaneous innovations maintain their cointegration relations. This implies high efficiency of contemporaneous information transmission across stock markets. Only when these two conditions are satisfied, stock markets are said to fully reflect new information in their own locations and across locations.

Appendix C

Figure C1 contains DAG results based on the innovations from our error correction model for all eleven stock markets for the four periods. A DAG shows the causal flow among a set of variables such that there are no directed cycles.[5] The nodes of these graphs represent variables on which data have been obtained while line segments connecting nodes (directed edges) are generated by calculations of conditional statistical dependence among pairs of variables (ceteris paribus). Under the assumption that variables $v_1, v_2, v_3, \ldots, v_n$ under study follow Markov processes, one can simplify the empirical joint distribution of these variables based on conditional statistical dependence.

Now we use X, Y, and Z to describe conditional statistical dependence among variables $v_1, v_2, v_3, \ldots, v_n$. For example, if there is a directed edge between variables X and Y like $X \rightarrow Y$, X is described as the parent of Y. In addition, a graph represented by $Y \leftarrow X \rightarrow Z$ implies that the three variable, X, Y and Z have a relation such that X causes Y and Z. This causal relationship implies that the unconditional association between Y and Z is nonzero but the conditional association between Y and Z , given the knowledge of the common cause X, is zero. Alternatively, a graph represented by $Y \rightarrow X \leftarrow Z$ implies that the unconditional association between Y and Z is zero but the conditional association between them, given the common effect X, is nonzero.

Following Pearl (2000), DAGs can be used to represent conditional independence as implied by the recursive product decomposition:

$$\Pr(v_1, v_2, \ldots, v_n) = \prod_{i=1}^{n} \Pr(v_i|\text{pa}_i), \tag{C-1}$$

where Pr is the probability of variables $v_1, v_2, \ldots, v_n$ and pa_i (also called parents) represents a set of variables that immediately causes v_i.

In Spirtes et al. (2000), a causal search algorithm, called the PC algorithm, is provided for making inference on directed acyclic graphs from observational data. It begins with a complete undirected graph, where every variable is connected to every other variable. Edges between variable are then removed based on vanishing correlation or partial correlation, at a predetermined level of significance. The significance level is a threshold for independence. The higher it is set, the less discerning the PC algorithm is when determining the indepen-

[5]This means that it is not possible to start at a variable and follow a directed path back to the same variable.

dence between two variables. Spirtes et al. (2000, p. 116) recommend that one drop the level of significance used as the number of observation increases. For small samples less than 100 observations, a significance level of 20% is recommended. For larger samples greater than 100 and less than 300 observations, they suggest a 10% significance level. In our research, because we have over two hundred observations for each period, we set the significance level at 10%. Therefore, if estimated correlations and partial correlations linking some variables that form edges are not statistically significantly different from zero at the 10% significance level, the causal search algorithm will remove those edges. The software TETRAD IV is employed to conduct the DAG analysis.

We apply the DAG to identify the dependence among the stock markets so that we can place zero restrictions on matrix $\mathbf{A}$ in our SVAR model. This strategy permits that restrictions imposed on matrix $\mathbf{A}$ can accurately reflect the data generating process. The PC algorithm sometimes generate graphs with cycles and bidirected edges, as shown in Panel B ($CN \leftrightarrow HK$) and Panel D ($CN \leftrightarrow HK$) of Figure C1. Since ignoring undirected edges might distort our SVAR analysis, both directions are considered for the undirected edge in the SVAR analysis with the level of significance set to 5%.

In $\mathbf{e}_t = \mathbf{A}^{-1}\mathbf{B}\mathbf{v}_t$ of our SVAR model, two 11×11 matrices $\mathbf{A}$ and $\mathbf{B}$ have to be estimated. Since $\mathbf{A\Sigma A'} = \mathbf{BB'}$, the expressions on both sides are symmetric. This fact imposes $11(11+1)/2$ restrictions on the 2×11^2 unknown elements in $\mathbf{A}$ and $\mathbf{B}$. Therefore, in order to identify $\mathbf{A}$ and $\mathbf{B}$, we need to supply at least $2 \times 11^2 - 11(11+1)/2 = 176$ additional restrictions. The parameter estimation of matrix $\mathbf{A}$ for the four periods are reported in Table C1. LR tests for over-identification are also reported in Table C1.

Figure C1: DAG-recovered patterns of contemporaneous shock transmission among eleven stock markets during four cointegration periods

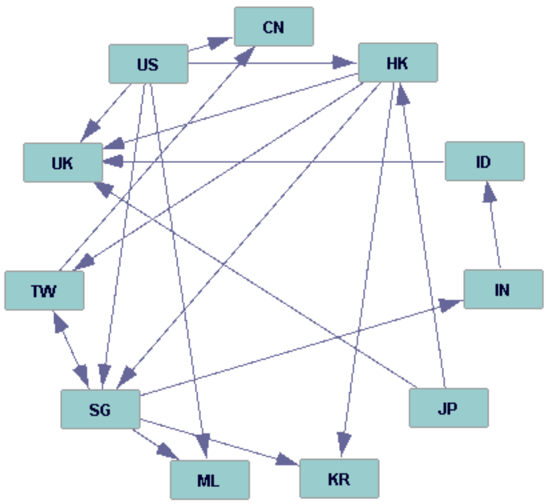

Figure C1: Panel A (Period 1)

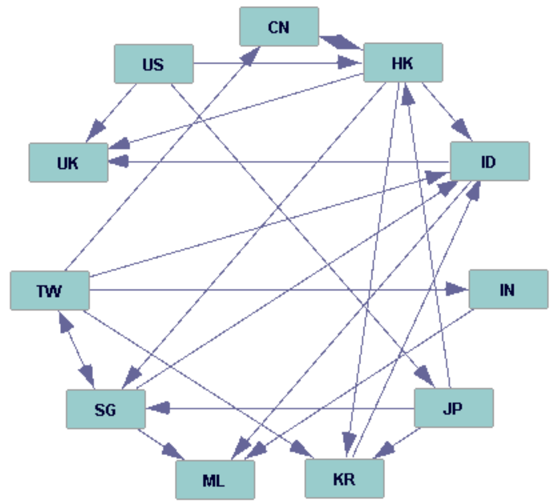

Figure C1: Panel B (Period 2)

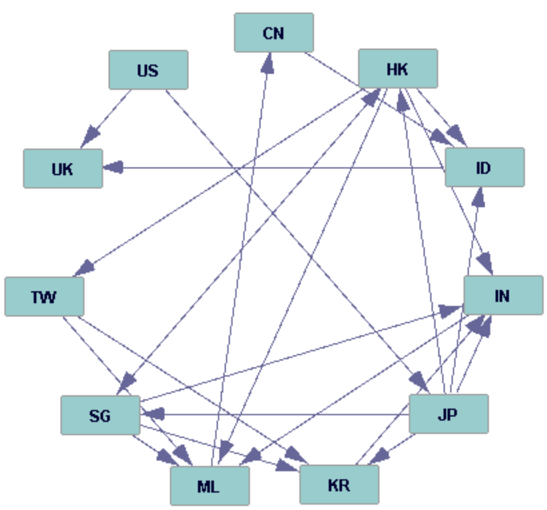

Figure C1: Panel C (Period 3)

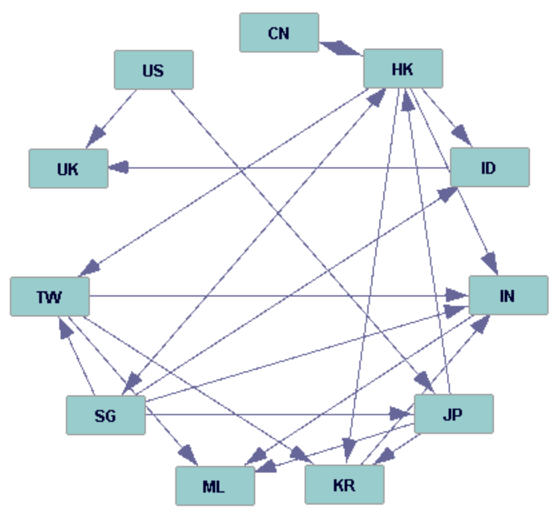

Figure C1: Panel D (Period 4)

Table C1: Parameter estimation of matrix **A** for four periods

$\mathbf{A}_{i,j}$	US	UK	JP	HK	TW	SG	KR	IN	ML	CN	ID
Panel A: Period 1 (Aug 24,1998-Aug 10,1999)											
US	1.0000										
UK	-0.3452 (0.0669)	1.0000	-0.1553 (0.0538)	-0.2149 (0.0443)							0.0928 (0.0422)
JP			1.0000								
HK	-0.6415 (0.1003)		-0.3784 (0.0847)	1.0000							
TW				-0.2831 (0.0603)	1.0000						
SG	-0.2093 (0.0872)			-0.5681 (0.0580)	-0.1692 (0.0615)	1.0000					
KR				-0.3308 (0.1197)		-0.2691 (0.1224)	1.0000				
IN						-0.6019 (0.0946)		1.0000			
ML	-0.6573 (0.1362)					-0.2125 (0.0883)			1.0000		
CN	0.1878 (0.0893)				-0.1610 (0.0652)					1.0000	
ID								-0.1724 (0.0464)			1.0000
Log likelihood:			5166.19								
LR test for over-identification:											
$\chi^2(37)$			41.907	[0.2664]							
Panel B: Period 2 (Sep 6,2001-Aug 26,2002)											
US	1.0000										
UK	-0.4777 (0.0820)	1.0000		-0.2812 (0.0919)							-0.1661 (0.0747)
JP	-0.4482 (0.0828)		1.0000								
HK	-0.2480 (0.0665)		-0.3743 (0.0547)	1.0000							
TW					1.0000						-0.4810 (0.0887)
SG			-0.1590 (0.0580)	-0.4293 (0.0772)	-0.1886 (0.0445)	1.0000					-0.1381 (0.0644)
KR			-0.3096 (0.0808)	-0.4383 (0.0968)	-0.4104 (0.0599)		1.0000				
IN					-0.1766 (0.0425)			1.0000			
ML						-0.3627 (0.0414)		-0.1055 (0.0454)	1.0000		-0.0797 (0.0396)
CN				-0.3021 (0.0890)	0.1787 (0.0618)					1.0000	
ID				-0.5489 (0.0832)			-0.0926 (0.0603)				1.0000
Log likelihood:											
LR test for over-identification:			5782.21								
$\chi^2(33)$			26.8752	[0.6298]							

Note: Parameter estimates of matrix **A** in the model $\mathbf{e}_t = \mathbf{A}^{-1}\mathbf{B}\mathbf{v}_t$ are reported in Panels A–D for different cointegration periods, respectively. The elements of **A** show contemporaneous correlations among observed residuals. The element (i, j) of matrix **A**, $\mathbf{A}_{i,j}$, gives how the observed residual of market i instantaneously responds to that of market j. Th table also reports the LR test results for over-identification. Standard errors are given in parentheses, while p-values are reported in brackets.

Table C1: Parameter estimation of matrix **A** for four periods—continued

$\mathbf{A}_{i,j}$	US	UK	JP	HK	TW	SG	KR	IN	ML	CN	ID
Panel C: Period 3 (Dec 21,2006-May 9,2008)											
US	1.0000										
UK	-0.4145 (0.0671)	1.0000									-0.2479 (0.0353)
JP	-0.8227 (0.0766)		1.0000								
HK			-0.9892 (0.0557)	1.0000							
TW				-0.4324 (0.0383)	1.0000						
SG			-0.3443 (0.0518)	-0.4411 (0.0388)		1.0000					
KR			-0.2945 (0.0521)		-0.2565 (0.0387)	-0.3409 (0.0513)	1.0000				
IN			0.2061 (0.0803)	-0.3599 (0.0656)		-0.4978 (0.0894)	-0.2833 (0.0810)	1.0000			
ML				0.1046 (0.0507)	-0.2112 (0.0426)	-0.3049 (0.0626)		-0.2364 (0.0445)	1.0000		
CN									-0.5800 (0.1244)	1.0000	
ID			-0.1975 (0.0775)	-0.6186 (0.0582)						0.0623 (0.0309)	1.0000
Log likelihood:											
LR test for over-identification:			8526.734								
$\chi^2(27)$			30.8331	[0.4500]							
Panel D: Period 4 (Oct 28,2008-Nov 20,2009)											
US	1.0000										
UK	-0.3606 (0.0526)	1.0000									-0.3222 (0.0442)
JP	-0.6522 (0.0634)		1.0000			0.0148 (0.0857)					
HK			-0.8587 (0.0741)	1.0000							
TW				-0.6434 (0.0400)	1.0000						
SG				-0.6194 (0.0453)	-0.2471 (0.0504)	1.0000					
KR			-0.2569 (0.0622)	-0.1803 (0.0662)	-0.4533 (0.0636)		1.0000				
IN				-0.2295 (0.0809)	-0.1994 (0.0745)	-0.2899 (0.0881)	-0.1373 (0.0671)	1.0000			
ML			-0.0964 (0.0265)		-0.1124 (0.0303)			-0.1870 (0.0297)	1.0000		
CN				-0.2890 (0.0710)					-0.3098 (0.1787)	1.0000	
ID				-0.2683 (0.1023)		-0.5011 (0.1145)					1.0000
Log likelihood:											
LR test for over-identification:			6472.153								
$\chi^2(33)$			32.6746	[0.3600]							

Note: Parameter estimates of matrix **A** in the model $\mathbf{e}_t = \mathbf{A}^{-1}\mathbf{B}\mathbf{v}_t$ are reported in Panels A–D for different cointegration periods, respectively. The entry A examines the contemporaneous correlation among observed residuals. The element (i, j) of matrix **A**, $\mathbf{A}_{i,j}$, gives how the observed residual of market i instantaneously responds to that of market j. The table also reports the LR test results for over-identification. Standard errors are given in parentheses, while p-values are reported in brackets.

Appendix D

In this appendix, we report the contagion measures for the four periods across all stock markets studied in this paper in Table D1. More specifically, the estimate contagion measures are reported in an 11×11 matrix for each cointegration period in Table D1. The element of the matrix, $C_{i,j}$, measures the contagion effect from a given market j (column) to another market i (row). In every case, the significance of the statistics is based on the Monte Carlo simulation method with 1000 replications.

Note that in Table D1 the elements in the first row, the second column and the diagonal are empty. As we define a trading day that starts from the U.S. and ends in the U.K., the markets that open after the U.S. market closes cannot affect the U.S. market. Hence, the contagion measures in the first row do not exist. By the same token, the U.K. market cannot affect other stock markets in the same trading day based on our trading day definition. Hence, the contagion measures in the second column do not exist either. Diagonal elements are empty as there are no contagion measures from a market to its own ($C_{j,j}$ is always equal to zero).

As shown in Table D1, little contagion effect is found between some Asian stock markets (Indonesia, Malaysia, China and India) for all cointegrated periods. Therefore, we focus on the $C_{i,j}$ values that are relevant to shocks from the U.S., Japan and Hong Kong markets.

To conduct a robustness test, we also use a different trading day definition, assuming that a trading day starts from the U.K. market and ends in the Asian markets. We find that this alternative trading day definition changes little to our analysis and conclusions, which are quite robust.

To provide a context, we also report the correlations among each and every pair of market portfolio index portfolio returns in Table D2.

Table D1: Estimates of contagion measures $C_{i,j}$ between stock markets for four periods

$\mathbf{C}_{i,j}$	US	UK	JP	HK	TW	SG	KR	IN	ML	CN	ID
Panel A: Period 1 (Aug 24,1998-Aug 10,1999)											
US											
UK	-0.0074		0.0373	0.0112	-0.0464	-0.0194	-0.0044	-0.0003	-0.0179	-0.0010	0.0079
JP	-0.0029			-0.0002	-0.0074	-0.0001	-0.0005	-0.0001	-0.0031	-0.0008	-0.0026
HK	0.0380		-0.0068		-0.0895	-0.0120	-0.0168	-0.0025	-0.0026	-0.0005	-0.0030
TW	**0.0324**		-0.0871	-0.1415		-0.1202	-0.0011	0.0000	-0.1436	-0.0002	-0.0009
SG	0.1080		-0.1074	-0.3160	-0.0245		-0.0003	0.0000	-0.0575	-0.0012	-0.0085
KR	-0.1151		0.0281	-0.3673	-0.0068	-0.1783		-0.0002	-0.0141	-0.0001	-0.0012
IN	0.0939		-0.0223	-0.1103	-0.0520	-0.1306	-0.0398		-0.0261	-0.0009	-0.0004
ML	-0.0813		-0.0014	0.0117	-0.1890	-0.2623	-0.0424	-0.0170		-0.0004	-0.0043
CN	-0.0811		-0.0148	-0.0508	0.0255	-0.0747	-0.0014	-0.0012	-0.0342		-0.0005
ID	-0.0209		0.0002	-0.0017	0.0002	-0.0127	-0.0001	-0.0238	-0.0040	-0.0005	
Mean	-0.0036		-0.0235	-0.0970	-0.0392	-0.0812	-0.0107	-0.0045	-0.0304	-0.0013	-0.0015
Panel B: Period 2 (Sep 6,2001-Aug 26,2002)											
US											
UK	-0.2302		0.0026	-0.0441	-0.0085	0.0000	-0.0008	-0.0960	-0.0135	-0.0035	0.0081
JP	0.0654			-0.0610	-0.0058	0.0000	-0.0045	-0.2966	-0.0549	-0.0101	-0.0890
HK	**0.1717**		0.0186		-0.1027	-0.0045	-0.0082	-0.1968	-0.0106	-0.0658	-0.0021
TW	0.0111		-0.0948	-0.2107		-0.1043	-0.0827	-0.7979	-0.7915	-0.0353	-0.5955
SG	0.0958		-0.0942	-0.1219	-0.1337		-0.0064	-0.2762	0.0000	-0.0202	-0.1381
KR	0.1335		-0.1787	-0.1025	-0.1100	-0.0050		-0.5587	-0.2201	-0.0009	-0.3151
IN	-0.0967		0.0003	-0.0071	**0.0307**	-0.0077	-0.0286		-0.5484	-0.0524	-0.1764
ML	-0.0033		-0.0099	0.0290	-0.0761	-0.0407	-0.0004	0.0111		-0.0158	-0.0298
CN	-0.2650		-0.0388	-0.0306	-0.2050	-0.0130	0.0000	-0.0570	-0.3055		-0.0304
ID	0.0688		-0.0353	-0.0571	-0.0401	0.0000	0.0048	-0.1374	-0.0136	-0.0141	
Mean	-0.0049		-0.0478	-0.0673	-0.0724	-0.0195	-0.0141	-0.2673	-0.2176	-0.0242	-0.1520
Panel C: Period 3 (Dec 21,2006-May 9,2008)											
US											
UK	-0.7976		0.0139	0.0222	-0.0289	-0.0123	0.0000	-0.0199	-0.0119	-0.0029	-0.0711
JP	-1.4624			-0.0659	-0.1562	-0.0617	-0.0085	-0.0118	-0.0166	-0.0010	-0.0361
HK	-1.0913		0.1773		-0.2358	-0.0571	-0.0142	-0.0548	-0.0288	-0.0025	-0.0960
TW	-0.2348		**0.1461**	0.0081		-0.1415	-0.1006	-0.0025	-0.0016	-0.0018	-0.6866
SG	-0.9672		**0.3093**	**0.1693**	-0.2432		-0.0239	-0.0428	-0.0385	-0.0001	-0.2930
KR	-1.4103		**0.2240**	**0.0668**	0.0642	-0.0531		-0.0273	-0.0455	-0.0001	-0.1700
IN	-2.3230		**0.3475**	**0.2140**	-0.2148	0.2020	-0.1403		-0.1195	-0.0057	-0.4823
ML	-0.6915		0.0252	0.0155	0.0449	-0.1018	-0.0136	**0.0540**		-0.0006	0.0000
CN	-1.8602		**0.0137**	-0.0325	-0.0892	0.0336	-0.0082	0.0001	-0.3973		-2.0686
ID	-2.3949		**0.4112**	**0.2315**	-0.4013	-0.2429	-0.1405	-0.0001	-0.1231	-0.0058	
Mean	-1.3233		0.1854	0.0699	-0.1400	-0.0483	-0.0500	-0.0117	-0.0870	-0.0023	-0.4337
Panel D: Period 4 (Oct 28,2008-Nov 20,2009)											
US											
UK	-0.1536		0.0048	0.0156	0.0011	-0.0079	-0.1217	-0.0008	-0.7890	-0.0062	0.0211
JP	-0.2843			0.0000	-0.0175	-0.0283	-0.0606	-0.0360	-1.0541	-0.0164	-0.0171
HK	-0.1981		0.0111		-0.0002	-0.1124	-0.1717	-0.0193	-0.9435	-0.0101	-0.0302
TW	0.0886		-0.0127	-0.2993		-0.1080	-0.0145	-0.0281	-0.6821	-0.0081	-0.0213
SG	-0.2200		**0.1475**	**0.2276**	0.0371		-0.1913	-0.0621	-0.7941	-0.0005	-0.0107
KR	-0.4203		**0.1489**	**0.1068**	0.0704	-0.0090		-0.2749	-1.4680	-0.0058	-0.0184
IN	0.0451		-0.1105	-0.3209	-0.0602	-0.3447	-0.1452		-0.3279	-0.0138	-0.1075
ML	-0.0223		-0.0603	-0.0480	-0.0044	-0.0145	0.0006	-0.0903		-0.0011	-0.0056
CN	0.0371		0.0410	-0.1421	-0.0120	-0.0112	0.0001	-0.0041	-0.2706		-0.0077
ID	-0.2630		**0.1458**	0.0562	-0.0143	**0.1128**	-0.1244	-0.0005	-0.3131	-0.0128	
Mean	-0.1391		0.0351	-0.0449	0.0000	-0.0581	-0.0921	-0.0573	-0.7381	-0.0083	-0.0219

Note: The contagion measure $C_{i,j}$ from market j (column) to market i (row) $\left(C_{i,j} = \left(\frac{\mathbf{\Phi}_{i,j} + \mathbf{\Phi}^*(0)_{i,j}}{\mathbf{\Phi}_{j,j} + \mathbf{\Phi}^*(0)_{j,j}} \right)^2 - \left(\frac{\mathbf{\Phi}_{i,j}}{\mathbf{\Phi}_{j,j}} \right)^2 \right)$ is reported for different cointegration periods. In every case, the significance of a contagion measure is based on the Monte Carlo simulation method with 1000 replications. The 5% quantile of $C_{i,j}$ that is greater than 0 (in bold font) indicates a significant contagion effect. The mean contagion measures are also reported for all markets and periods.

Table D2: Correlations among the eleven stock market portfolio daily returns

	US	UK	JP	HK	TW	SG	KR	IN	ML	CN	ID
Panel A: Correlation matrix, July 3,1997-April 30,2014											
US	1.0000										
UK	0.3977	1.0000									
JP	0.5096	0.4036	1.0000								
HK	0.4919	0.4717	0.5551	1.0000							
TW	0.3864	0.3088	0.4497	0.4996	1.0000						
SG	0.4195	0.4490	0.5099	0.7240	0.5077	1.0000					
KR	0.3727	0.3440	0.4888	0.5369	0.4854	0.4997	1.0000				
IN	0.2906	0.2741	0.3586	0.5043	0.3576	0.5191	0.3914	1.0000			
ML	0.2824	0.2277	0.2817	0.4403	0.2927	0.4631	0.3224	0.4250	1.0000		
CN	0.1496	0.1039	0.1904	0.2750	0.1757	0.1997	0.1343	0.1673	0.1329	1.0000	
ID	0.2666	0.3341	0.3392	0.4610	0.3261	0.4470	0.3543	0.3713	0.2435	0.1767	1.0000
Mean	0.3567	0.3241	0.3967	0.4916	0.3575	0.4257	0.3006	0.3212	0.1882	0.1767	
Panel B: Correlations, December 21,2006-May 9,2008											
US	1.0000										
UK	0.3567	1.0000									
JP	0.4361	0.5147	1.0000								
HK	0.4952	0.5631	0.8195	1.0000							
TW	0.5683	0.4505	0.6258	0.6825	1.0000						
SG	0.5021	0.6265	0.7489	0.8764	0.6721	1.0000					
KR	0.4465	0.6188	0.7805	0.8025	0.7031	0.8005	1.0000				
IN	0.4798	0.5945	0.6850	0.8406	0.6410	0.8674	0.7781	1.0000			
ML	0.5912	0.4881	0.6119	0.7521	0.7511	0.7736	0.7011	0.8290	1.0000		
CN	0.2681	0.2270	0.3478	0.4762	0.4116	0.3587	0.2818	0.3671	0.3898	1.0000	
ID	0.4012	0.5382	0.6479	0.7686	0.5955	0.7154	0.6668	0.7550	0.6913	0.4592	1.0000
Mean	0.4545	0.5135	0.6584	0.7427	0.6291	0.7031	0.6070	0.6504	0.5405	0.4592	

Note: The correlations are calculated based on the daily market portfolio return data for the U.S. (US), U.K. (UK), Japan (JP), Hong Kong (HK), Taiwan (TW), Singapore (SG), Korea (KR), Indonesia (ID), Malaysia (ML), China (CN), and India (ID). The U.S. data at $t-1$ are aligned with the data of other countries at t due to the selected time zone order from the U.S. stock market, to the Asian stock markets, and, then, to the U.K. stock market.