A Portfolio Optimization Model with Regime-Switching Risk Factors for Sector Exchange Traded Funds*

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Abstract

This paper develops a portfolio optimization model with a market neutral strategy under a Markov regime-switching framework. The selected investment instruments consist of the nine sector exchange traded funds (ETFs) that represent the U.S. stock market. The Bayesian information criterion is used to determine the optimal number of regimes. The investment objective is to dynamically maximize the portfolio alpha (excess return over the T-Bill) subject to neutralization of the portfolio sensitivities to the selected risk factors. The portfolio risk exposures are shown to change with various style and macro factors over time. The maximization problem in this context can be established as a regime-dependent linear programming problem. The optimal portfolio constructed as such is expected to outperform a naive benchmark strategy, which equally weights the ETFs. We evaluate the in-sample and out-of-sample performance of the regime-dependent market neutral strategy against the equally weighted strategy. We find that the former generally outperforms the latter.

1 Introduction

The goal of investment is to maximize the expected return of an asset portfolio with limited risk exposure. This investment objective is usually constrained with the changing economic conditions and the state-dependent risk factors. Dynamic asset allocation is a process of selecting investment instruments and constructing optimal portfolios over time.

The performance of any investment portfolio depends on the accuracy of forecast of asset returns and relevant risk factors. Chen, Ross and Roll (1986) studied an asset pricing model using macro economic risk factors such as the growth rate of industrial production (IP), unexpected inflation (UI), change of expected inflation (DEI), yield spread (YS) and credit spread (CS). They found that each factor is significant in predicting stock returns. Fama and French (1993) investigated an asset pricing model using three style factors: the excess market portfolio return over the risk free asset (MKT), difference of returns on the small and large stock portfolios (SML), and difference of returns on high and low book-to-market ratio portfolio returns (HML). However, neither Chen, Ross, and Roll (1986) nor Fama and French (1993) addressed the regime-dependent nature of the sensitivities of asset returns to the risk factors. In this paper, we develop an investment model incorporating market regimes which characterize different patterns of asset returns in the unobservable economic situations, such as bear and bull markets.

Investment securities may exhibit different risk levels in different economic situations and, therefore, different risk premiums. However, there is no clear determination as to which economic regime we are in by directly observing the market data. The key idea for a regime switching model is to resolve the issue of unobserved economic regimes over time. Previous research has indicated that a probability distribution with a structural Markov chain is sufficient to describe the dynamics of the economic regimes. Hamilton (1989) successfully applied a two-regime hidden Markov model to the U.S. GDP data and characterized the changing pattern of the US economy. Cai (1994), Hamilton (1998), and Gray (1996) used variations of the Markov regime-switching model to describe the time series behavior of U.S. short-term interest rates. Bekaert and Hodrick (1993) documented regime shifts in major foreign exchange rates. Schwert (1989) considered that asset returns may be associated with either high or low volatility regimes which switch over time. Whitelaw (2001) constructed an equilibrium model where growth in consumption follows a regime-switching process. Liu, Xu and Zhao (2011) showed that the regime-switching model is an effective way for linking sector ETF returns to style and macro factors in changing market regimes over time.

With new empirical evidence supporting regime-switching asset pricing models, regime-dependent asset allocation appears to be a flexible and attractive option to investors when market regimes can be properly identified. Ang and Bekaert (2002, 2004) studied asset allocation models with regime shifts. Guidolin and Timmermann (2007, 2008) provided important economic insights on how investments vary across different market regimes. Recently, Jun Tu (2010) provided a Bayesian framework for making portfolio decisions with regime-switching and asset pricing model uncertainty. Berdot, Goyeau and Leonard (2006) studied multi-sector portfolio allocation for active portfolio management and found that portfolio returns are very different with four market regimes.

A standard asset pricing model linearly relates the expected returns to market risk factors. Under a linear asset pricing model, such as the Capital Asset Pricing Model, the asset "alpha" is usually referred to as the temporary deviation of the expected returns from the prediction of the pricing model. Portfolio managers usually pursue a market neutral strategy so that the portfolio's alpha is maximized with neutralized risk exposures.[1] This investment strategy is often used in the hedge fund industry. According to Capocci (2006), approximately 28.3% of the MAR/CISDM (Global Management Account Reports/Center for International Securities and Derivatives Markets) individual funds in the database are market neutral funds. Edwards and Caglayan (2001) showed that market neutral strategies provide the investor with positive returns in the market downturns during the period from 1990 to 1998.

In this paper, we propose a stochastic linear programming model to maximize the portfolio "alpha" with limited risk exposure to the selected risk factors. According to Shyu et al. (2006), this feature is also referred to as the zero-beta strategy. As pointed out by Gastineau, Olma and Zielinski (2007), this market neutral strategy is implemented by constructing and rebalancing the portfolio that has overall zero betas for all relevant risk factors and thus the return of the portfolio under such strategy is

[1]The standard feature of the market neutral strategy is that, while some assets in a portfolio have long positions, some assets have short positions simultaneously. In this way the impact of market movements can be minimized. In a declining market situation, the short positions earn profits by neutralizing the losses made by long positions. By taking long positions on undervalued assets and short positions on overvalued assets, steady returns can be captured in all directions of market.

uncorrelated with the market risk factors.

One of the key equity investment trends in recent years is the growing interest in the exchange traded funds (ETFs). Unlike traditional mutual funds, these instruments are exchange-tradable securities like normal stocks. Investing in a sector ETF is an investment strategy which mimics the performance of the corresponding industrial sector. In this paper, an optimization problem is set up for finding the optimal investment weights in the nine sector ETFs that represent the U.S. stock market based on a regime-dependent market neutral strategy. The performance of the portfolio under the regime-dependent market neutral strategy is then compared with that of the portfolio under a benchmark strategy, which allocates assets equally across the nine U.S. sector ETFs regardless of the future market regime. The empirical results demonstrate that the regime-dependent market neutral strategy generally outperforms the benchmark strategy.

2 The Asset Pricing Model

In the financial market, there are many broad asset classes (such as equities, bonds, commodities, currencies, and real estate properties). Asset allocation is made among these broad asset classes and there are possibly many different strategies. In this research, we only consider equity instruments in the form of the nine U.S. sector ETFs. The Sector SPDRs ETFs cover all sectors of the U.S. stock market such as consumer discretionary (XLY), consumer staples (XLP), energy (XLE), financials (XLF), health (XLV), industrials (XLI), materials (XLB), technology (XLK), and utilities (XLU). These sector ETFs satisfy four selection criteria: (i) the nine sector ETFs well represent the major sectors of the U.S. stock market; (ii) the nine sector ETFs are offered by one company so as to maintain portfolio consistency and eliminate managerial discrepancies; (iii) the nine sector ETFs have a long trading history (started December 23, 1998); and (iv) the nine sector ETFs are liquid and have a large daily trading volume. The returns on assets are considered for fixed time periods.

2.1 The basic models

The models for asset returns and investment decisions will be formulated in general and then specialized to ETFs in the application. Let P_{ti} be the trading price of asset i at time t, with $R_{ti} = \ln(P_{ti}/P_{t-1,i})$ being the return for the ith ($i = 1, \cdots, I$) asset in period t ($t = 1, \cdots, T$). The term return will henceforth refer to the logarithm of gross return. R_t is the vector of the asset returns in period t. It is assumed that the financial market in each period can be realized as one of N regimes, with the statistical distribution of asset returns depending on the regime. Furthermore, the regimes are characterized by a set of J risk factors, which represent broad macro and micro economic indicators. Let F_{tj} be the value of the jth risk factor ($j = 1, \cdots, J$) in period t. Correspondingly, F_t is the vector of risk factors in period t. Asset returns in different market regimes are characterized with the common risk factors F_t.

Suppose that the market is in regime s_t in period t and consider that the asset returns are defined by the regime-dependent linear factor model

$$R_t = A_{s_t} + B_{s_t} F_t + \Gamma_{s_t} e_t, \tag{1}$$

where $e_t \sim N(0, I)$. The model parameters $\{A_{s_t}, B_{s_t}, \Gamma_{s_t}\}$ depend on the regime s_t. The vector $A_{s_t}' = (\alpha_{1s_t}, \cdots, \alpha_{Is_t})$ contains the state-depend intercepts of the linear factor model. The matrix

$$B_{s_t} = \begin{pmatrix} \beta_{11s_t} & \dots & \beta_{1Js_t} \\ \vdots & \ddots & \vdots \\ \beta_{I1s_t} & \dots & \beta_{IJs_t} \end{pmatrix}$$

defines the sensitivities of asset returns to the common risk factors in state s_t. One implication of the linear factor model is that the conditional asset returns within a regime, given the factors, are normally distributed with mean vector $\mu_{s_t} = A_{s_t} + B_{s_t} F_t$ and covariance matrix $\Sigma_{s_t} = \Gamma_{s_t} \Gamma_{s_t}'$.

The applicability of the linear factor model for predicting returns in a time period requires estimates of the regime-dependent parameters and forecasts for the values of the factors. Assume that there exist N distinct regimes and the dynamics of the market regimes follow a Markov chain. With the regimes process $\{s_t, t = 0, 1, \ldots\}$, consider that the regimes are indexed by n and $q_{tn} = Pr[s_t = n]$, $n = 1, \ldots, N$. There is an initial regime distribution q_0 and a transition probability matrix $P = \{p_{mn}\}$, where the transition probability from regime m to regime n is given by

$$p_{mn} = Pr(s_{t+1} = n | s_t = m), \forall m, n. \tag{2}$$

The conditional returns R_n for regime n have a normal density $R_n \propto f_n(r)$. The unconditional distribution of the asset returns in period t given regime m in period

$t-1$ is a mixture of normal distributions: $f(r) = \sum_{n=1}^{N} p_{mn} f_n(r)$, which is able to capture distributional characteristics such as heavy tails. In period t, given regime m in period $t-1$, the unconditional expected asset return is

$$\bar{\mu}_{tm} = \sum_{n=1}^{N} \mu_{tn} p_{mn},$$

and the covariance matrix is

$$\bar{\Sigma}_{tm} = \sum_{n=1}^{N} [(\mu_{tn} - \bar{\mu}_{tm})^2 + \Gamma_n \Gamma'_n] p_{mn}.$$

If the regimes are known in each period, then the estimation of model parameters from observations on returns and factors is straightforward. However, the regime in each period is in fact unknown, as are the model parameters for each regime. The regime must be inferred and the model parameters must be estimated from data.

2.2 The estimation algorithm

An established estimation procedure for identifying regimes and estimating parameters is the EM algorithm (see Dempster et al. (1977)). The EM algorithm consists of two steps. The E-step is the estimation of the missing data for regimes and the M-step is the maximization of the likelihood based on the estimated missing data on regimes. The EM algorithm requires the specification of the number of regimes. The algorithm is augmented with a third step, called the N-step, for the determination of the number of regimes based on the Bayesian information criterion developed by Schwarz (1978).

Denote the model parameters as $\theta = \{A_{s_t}, B_{s_t}, \Gamma_{s_t}, q_0, P\}$, the unknown regimes at each time as S, and the observed data on returns and factors as X. The iterative algorithm can be designed as follows:

The E-step: Set an initial value θ^0 for the true parameter set θ, and calculate the conditional distribution for regimes, $Q(S) = P(S|X;\theta^0)$. Determine the expected log-likelihood of the data with respect to the regimes, $E_Q[\ln P(X, S; \theta)]$.

The M-step: Maximize the expected log-likelihood with respect to the conditional distribution of the hidden regimes to obtain an improved estimate of θ. The improved estimate is

$$\theta_1 = \arg\max_{\theta} \{E_Q[\ln P(X, S; \theta)]\}. \tag{3}$$

With θ_1 as the new value for θ, return to the E-step.

The outputs from the EM algorithm are: (a) parameter estimates

$$\hat{\theta} = \{(\hat{A}_n, \hat{B}_n, \hat{\Gamma}_n, \forall n = 1, \ldots, N)\},$$

(b) the estimated transition matrix $\hat{P}$, and (c) the posterior distribution of regimes. The implied regime, $\hat{S}$, at each time is the most likely regime.

The EM algorithm requires a known number of regimes, which must be determined from the data. The objective is to find the best fitting model and the number of regimes is part of the fit. A third step in the estimation is identifying the optimal number N^* of regimes under the Bayesian information criterion (BIC).

The N-step: Let $\hat{\theta}(N)$ be the parameter estimates with N regimes, and $L^*(N)$ be the maximized value of the likelihood function. If T is the number of data points and $Z(N)$ is the number of free parameters in the N regime model, then the Bayesian information criterion (BIC) is a penalized likelihood defined by

$$\text{BIC}(N) = -2 \ln L^*(N) + Z(N) \ln(T). \tag{4}$$

Then

$$N^* = \arg\min \ \text{BIC}(N). \tag{5}$$

An important ingredient in predicting the asset returns with the linear model is the value for the vector of factors. The factors are expected to characterize the market regimes, so that the pattern in factors is regime dependent. It will be assumed that the regime pattern in factors is stationary. So transition to a regime implies a factor pattern and a relationship of asset returns to those factors.

3 A Regime-Dependent Market Neutral Strategy

With forecasts for the distribution of returns, the objective is to allocate investment capital to risky assets so that investor goals are attained. The goals are typically stated in terms of return and risk. For the linear factor model of asset returns defined above, the measure of excess return is the expected "alpha" of the portfolio and the measure of risk is the regime dependent "beta" of the portfolio.

Unlike the Markowitz (1952) mean-variance model or a standard utility maximization model, our focus is to maximize portfolio alpha with risk exposure constraints. With a planning horizon T, investment decisions are made in each time period t, $0 \leq t \leq T$. Let w_{ti} be the portfolio weight (fraction of investment capital) in asset i in period t, where $\sum_{i=1}^{I} w_{ti} = 1$. Transactions costs from portfolio rebalancing are not considered.

If the regime in period $t-1$ is m and the portfolio weights for period t is $w_t^{'} = (w_{t1}, \cdots, w_{tI})$, then the one-period expected portfolio alpha is

$$\Psi_m(w_t) = E[A_{s_t}^{'} w_t | s_{t-1} = m] = \sum_{n=1}^{N} \sum_{i=1}^{I} w_{ti} \alpha_{in}\, p_{mn} \tag{6}$$

Hence, the unconditional alpha with respect to the posterior probabilities of the regimes can be calculated upon obtaining the new information at each decision point in time.

To control for systematic risk, constraints are placed on the regime-dependent portfolio beta. Although risk neutrality is desired, taking some risk could lead to considerable gains in returns. So a regime-dependent risk tolerance parameter δ is introduced to permit a limited exposure to the common risk factors. In each possible regime in the next period, the portfolio beta for factor j in regime n is defined as

$$\Phi_{jn}(w_t) = \sum_{i=1}^{I} w_{ti} \beta_{ijn}, \quad \forall j = 1, \cdots, J, \text{ and } n = 1, \cdots, N. \tag{7}$$

Thus, the portfolio risk exposure is constrained as

$$-\delta_n \leq \Phi_{jn}(w_t) \leq \delta_n, \quad \forall j = 1, \cdots, J, \text{ and } n = 1, \cdots, N. \tag{8}$$

The tolerance parameter δ could depend on the particular factor and regime, but a common tolerance is used here.

Although short sales are permitted, a limit is placed on the fraction of shorts. A maximum fraction of the long position in each of the assets is also imposed. These constraints are

$$-\xi_l \leq w_{ti} \leq \xi_u. \tag{9}$$

where $\xi_l \geq 0$ and $\xi_u \geq 0$.

With the reformulation of the objective and constraints, the portfolio optimization for period t is determined from the following stochastic linear programming problem:

$$
\begin{aligned}
\max_{w_t} \quad & \Psi_m(w_t) \\
\text{s.t.} \quad & \Phi_{jn}(w_t) \leq \delta_n, \quad \forall j = 1, \cdots, J, \text{ and } n = 1, \cdots, N, \\
& \Phi_{jn}(w_t) \geq -\delta_n, \quad \forall j = 1, \cdots, J, \text{ and } n = 1, \cdots, N, \\
& \sum_{i=1}^{I} w_{ti} = 1, \\
& -\xi_l \leq w_{ti} \leq \xi_u, i = 1, \cdots, I.
\end{aligned} \tag{10}
$$

There are important implications for investment decisions following from the structure assumed for asset returns. In each period, there exists an unknown regime and a returns distribution for assets conditional on the regime. Also the transitions between regimes are Markovian, with a constant transition probability matrix. The investment decision is to be made at the beginning of each period, given the regime in the prior period and the chance of switching to each of the possible regimes in the current period.

4 Application to Exchange Traded Funds

The single period investment model for a regime-dependent market neutral strategy is now applied to market data. The risky financial instruments considered for investment are the S&P Sector ETFs (SPDRs). The nine ETFs are listed in Table 1.

The Sector SPDRs are unique ETFs that divide the S&P 500 into nine sector index funds. Together, the nine Sector SPDRs represent the S&P 500 as a whole. The Sector SPDRs let the investor achieve the security of investing in the well-known, large cap stocks of the S&P 500, with the ability to over-weight or under-weight particular sectors based on investment goals and strategies.

The Sector SPDRs satisfy the four selection criteria discussed previously. The returns on assets are considered to be dependent on regimes which are in turn defined by market conditions. A number of factors have been found to have a significant effect on returns. Table 2 lists important factors found in the literature (Fama and French (1993), Chen, Roll and Ross (1986), and Schaefer and Strebulaev (2008)). Daily returns on the ETFs from January 3, 2005 to September 30, 2009 were retrieved from Bloomberg. For the same period the style and macro factors data were retrieved from French data library (MKT, SMB, and HML) and Datastream (VIX, YS, and CS). The

Table 1: Assets—Exchange Traded Funds of the U.S. Stock Market

Fund	Asset Class	Description
Cons D	Consumer Discretionary	The group includes McDonald's, Walt Disney Co., and Comcast.
Cons S	Consumer Staples	Component stocks include Wal-Mart, Proctor & Gamble, Philip Morris International, and Coca-Cola.
Enr	Energy	Leaders in the group include ExxonMobil Corp., Chevron Corp., and ConocoPhillips.
Fin	Financials	Among the companies included in the group are JPMorgan Chase, Wells Fargo, and BankAmerica Corp.
Hlth	Health	Pfizer Inc., Johnson & Johnson, and Abbott Labs are included in this group.
Ind	Industrials	General Electric Co., Minnesota Mining & Manufacturing Co., and United Parcel are among the largest components by market capitalization in this sector.
Mat	Materials	Among its largest components are Monsanto, E.I. DuPont de Nemours & Co., and Dow Chemical Co.
Tech	Technology	Components include Microsoft Corp., AT&T, International Business Machines Corp., and Cisco.
Utl	Utilities	The component companies include Exelon Corp., Southern Co., and Dominion Resources Inc.

days from January 3, 2005, to February 26, 2009, constitute the in-sample data for estimation and the strategy determination. The out-of-sample data for model evaluation covered February 27 to September 30, 2009.

Table 2: Market Factors

Factor	Definition
MKT	Value weighted returns on all NYSE, AMEX, NASDAQ stocks minus 30- day US T-Bill yield
SMB	Differential returns between small and large cap stock portfolios
HML	Differential returns between high and low book-to-market stock portfolios
VIX	Weighted blend of implied volatility estimates for options on S&P 500
YS	Difference between yields of 20-year Treasury bond and 3-month US T-Bill.
CS	Difference between yields of top rated bond and lowest grade bond of same maturity

4.1 Market regimes

The number of regimes is the starting point for the regime-switching linear factor model for asset returns. The Bayesian information criterion (BIC) corresponding to maximum likelihood estimation for a given number of regimes is plotted in Figure 1. The optimal number of regimes is determined to be 3 by the BIC.

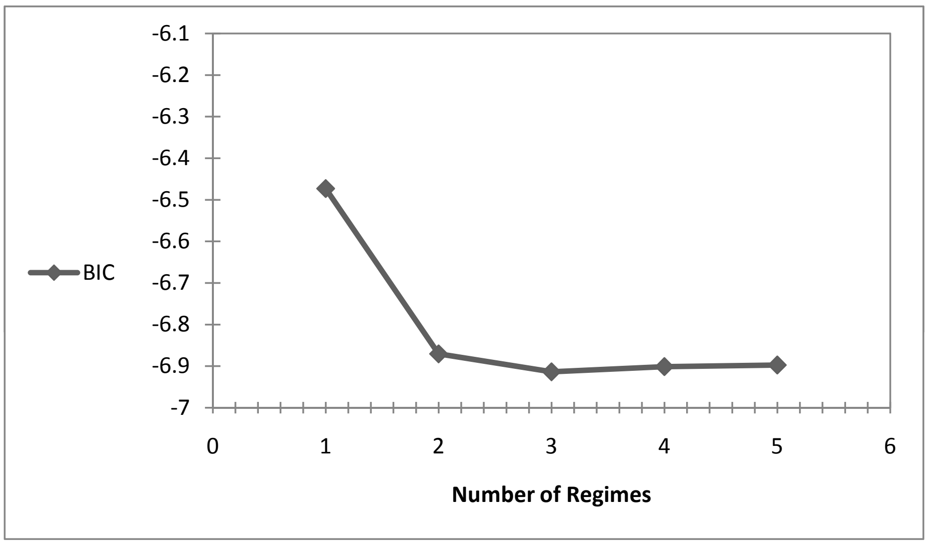

Figure 1: BIC by Number of Regimes

The estimation with 3 regimes provides a classification of time periods into three regimes and a probability matrix for transition among the three regimes. The regimes are characterized by the common risk factors, so the sample means of these factors by regime should assist us in interpreting each regime. In Table 3 are average daily percent changes in the common risk factors by regime.

Table 3: Sample Factor Means across Regimes

Regime	MKT	SMB	HML	VIX	YS	CS
1	-0.3262	0.0304	-0.1440	0.15	2.7255	2.6438
2	-0.1131	-0.0117	-0.0022	0.11	1.6049	1.3438
3	0.0404	-0.0048	0.0211	0.01	0.4618	0.8804

Table 3 shows that regime 1 can be described as a "bear" market with a declining market index and increasing spreads. Regime 3 is a "bull" market as the market index is growing and the spreads are small. Regime 2 is between the two other regimes and can be classified as a "transition" market.

The transition probability matrix for the 3 regime model is estimated by

$$\hat{P} = \begin{pmatrix} 0.8933 & 0.1067 & 0.0000 \\ 0.0455 & 0.9346 & 0.0199 \\ 0.0000 & 0.0098 & 0.9902 \end{pmatrix}.$$

The matrix shows that regimes are stable, and switching occurs with low probability.

4.2 Alpha and beta estimates

For the 3 regime specification the estimated parameters for the regime-switching linear factor model relating asset returns to factors are provided in Table 4. The most important observation is that the "alphas" and "betas" are not fixed but vary across the regimes. There is more potential for excess returns (alphas) in the bull regime, so it is expected that advance knowledge of the future regime will affect the investment decision on how to allocate assets across nine ETFs.

4.3 Portfolio weights

Using the estimates for alphas and betas, the stochastic linear programming problem is solved for the portfolio weights applicable to the next period (day). The limits on investment fractions $\xi_l = -0.5$ and $\xi_u = 0.5$ are imposed. This allows for a lot of variation in the allocation of investment capital to the asset classes. At each decision point, the starting market regime is inferred based on the maximum posterior probability of the regimes, so the optimal portfolio weights are given by the starting regime. As the level of tolerance δ varies, the regime-dependent market neutral strategy sug-

Table 4: Alpha and Beta Estimates by Regime

Regime 1		Beta					
Asset	Alpha	MKT	SMB	HML	VIX	YS	CS
Cons D	0.0072	0.0097	0.0038	0.0021	0.0510	-0.0045	0.0019
Cons S	-0.0071	0.0049	-0.0002	-0.0010	0.0066	0.0010	0.0009
Enr	0.0015	0.0106	0.0014	0.0189	-0.0715	-0.0016	0.0006
Fin	0.0078	0.0133	-0.0055	-0.0021	-0.0923	0.0024	-0.0049
Hth	-0.0043	0.0066	-0.0001	-0.0035	-0.0027	-0.0004	0.0013
Ind	0.0004	0.0087	0.0009	-0.0016	-0.0033	-0.0022	0.0010
Mat	-0.0010	0.0119	0.0007	-0.0047	0.0812	-0.0021	0.0017
Tech	-0.0017	0.0094	-0.0016	-0.0024	-0.0292	-0.0013	0.0016
Util	-0.0180	0.0074	-0.0036	-0.0049	-0.0708	0.0039	0.0021

Regime 2		Beta					
Asset	Alpha	MKT	SMB	HML	VIX	YS	CS
Cons D	-0.0008	0.0101	0.0019	0.0040	-0.0143	0.0003	-0.0001
Cons S	0.0005	0.0036	-0.0003	0.0007	-0.0651	0.0001	0.0000
Enr	0.0057	0.0128	-0.0076	-0.0090	0.0246	0.0030	-0.0080
Fin	-0.0055	0.0132	0.0020	0.0154	-0.0863	-0.0036	0.0080
Hth	0.0003	0.0041	0.0001	0.0001	-0.0757	-0.0006	0.0007
Ind	0.0005	0.0096	-0.0005	0.0007	-0.0192	0.0007	-0.0013
Mat	0.0021	0.0123	-0.0002	-0.0065	0.0090	0.0010	-0.0023
Tech	-0.0005	0.0098	-0.0011	-0.0032	0.0103	0.0016	-0.0015
Util	0.0014	0.0047	-0.0035	-0.0020	-0.0615	-0.0005	-0.0002

Regime 3		Beta					
Asset	Alpha	MKT	SMB	HML	VIX	YS	CS
Cons D	-0.0027	0.0096	0.0002	-0.0031	-0.0105	0.0001	0.0029
Cons S	-0.0006	0.0061	-0.0024	-0.0027	-0.0201	0.0001	0.0007
Enr	0.0054	0.0161	0.0011	0.0252	0.0488	-0.0002	-0.0059
Fin	-0.0006	0.0090	-0.0032	-0.0014	-0.0353	0.0000	0.0006
Hth	-0.0010	0.0075	-0.0034	-0.0060	-0.0055	0.0004	0.0011
Ind	-0.0033	0.0090	-0.0004	-0.0029	-0.0137	0.0003	0.0038
Mat	0.0033	0.0121	0.0024	0.0045	-0.0112	-0.0007	-0.0033
Tech	-0.0007	0.0089	-0.0009	-0.0088	-0.0259	-0.0001	0.0012
Util	0.0038	0.0101	-0.0027	0.0101	0.0147	-0.0005	-0.0040

gests different long and short positions for each regime. Tables 5, 6, and 7 provide the optimal portfolio weights for sector ETFs for the bull, transition, and bear markets.

Table 7 provides the optimal weights for regime 3 (bear market) with the tolerance level (δ) at 0.01 and 0.03, as the weights at the tolerance level at 0.05 are the same as those with the tolerance level at 0.03. That is, the optimal weights with the tolerance

Table 5: Optimal Weights for Regime 1 (Bull Market)

Asset	Tolerance Level(δ)		
	0.01	0.03	0.05
Consumer Discretionary	0.5000	0.5000	0.5000
Consumer Staples	0.0961	-0.5000	-0.5000
Energy	0.3595	0.5000	0.5000
Financials	0.3581	0.5000	0.5000
Health	0.5000	-0.1738	-0.4064
Industrials	0.5000	0.5000	0.5000
Material	-0.5000	0.5000	0.5000
Technology	-0.3137	-0.3262	-0.0936
Utility	-0.5000	-0.5000	-0.5000

Table 6: Optimal Weights for Regime 2 (Transition Market)

Asset	Tolerance Level(δ)		
	0.01	0.03	0.05
Consumer Discretionary	-0.5000	-0.5000	-0.5000
Consumer Staples	0.5000	0.5000	0.5000
Energy	0.4632	0.5000	0.5000
Financials	-0.5000	-0.5000	-0.5000
Health	0.5000	0.1725	0.0000
Industrials	0.5000	0.5000	0.5000
Material	0.0636	0.5000	0.5000
Technology	0.3633	-0.2139	-0.5000
Utility	-0.3901	0.0414	0.5000

Table 7: Optimal Weights for Regime 3 (Bear Market)

Assets	Tolerance Level(δ)	
	0.01	0.03
Consumer Discretionary	-0.5000	-0.5000
Consumer Staples	0.5000	0.5000
Energy	0.1728	0.5000
Financials	-0.3277	0.5000
Health	0.1548	-0.5000
Industrials	-0.5000	-0.5000
Material	0.5000	0.5000
Technology	0.5000	0.5000
Utility	0.5000	0.5000

level at 0.03 is the corner solution and further slacks given by a higher tolerance level do not change the solutions.

As the risk tolerance is relaxed the portfolio weights move to the limits. In fact, for the transition and bear markets the boundary solution is optimal for lower tolerance levels. It is in the bull market, where the trade-off between risk and return is greater, that solutions are more sensitive to the beta tolerance level. In each of the regimes short selling some sector ETFs provides capital to invest in more promising sector ETFs.

5 Portfolio Performance

The regime-dependent strategies will be implemented for the ETFs during the period from February 27, 2009 to September 30, 2009, a period of 150 trading days. At the start of a day the implied regime (from the Viterbi (1967) algorithm) is considered to be the true regime and the regime-dependent strategy is implemented. This strategy is then compared with the benchmark strategy, a popular approach to diversification with fixed portfolio weights over the investment horizon. Wealth is accumulated from the actual daily returns for ETFs during the study period.

Table 8 shows the mean, variance, standard deviation and Sharpe ratio of portfolio returns under the regime-dependent strategy and benchmark strategy for the period from February 27, 2009 to September 30. 2009.

For mean returns (either excess returns or cumulative returns), the portfolio performance under the regime-dependent strategy dominates that of the benchmark strategy, but with greater standard deviation (risk).

By examining the Sharpe ratio, we can get a picture of excess returns per unit of risk. The Sharpe ratio I in Table 8 is defined by the difference of portfolio returns under the regime-dependent strategy and the benchmark strategy divided by the standard deviation of the difference of these returns. These Sharpe ratios are all positive and suggest that the returns earned by the regime-dependent strategy are not due to excess risk.

The Sharpe ratio II is defined by the difference of mean returns between the portfolio and a three month T-Bill yield divided by the standard deviation of the portfolio return. For either excess return or cumulative return, the Shape ratio II of the portfolio the under regime-dependent strategy is much higher than that of the benchmark strategy.

The regime in each day is implied by the analysis, and the performance in each

Table 8: Overall Performances

Panel A Tolerance Level=0.01				
	Excess Return		Cumulative Return	
	Portfolio	Benchmark	Portfolio	Benchmark
Mean	0.0032	0.0024	1.0904	1.0752
Variance	0.0007	0.0003	0.0134	0.0073
Standard Deviation	0.0273	0.0173	0.1160	0.0853
Sharpe Ratio (I)	0.0530		0.3780	
Sharpe Ratio (II)	0.0551	0.0388	0.1086	-0.0304
Panel B Tolerance Level=0.03				
	Excess Return		Cumulative Return	
	Portfolio	Benchmark	Portfolio	Benchmark
Mean	0.0050	0.0024	1.1404	1.0752
Variance	0.0015	0.0003	0.0465	0.0073
Standard Deviation	0.0390	0.0173	0.2156	0.0853
Sharpe Ratio (I)	0.1050		0.4480	
Sharpe Ratio (II)	0.0846	0.0388	0.2902	-0.0304
Panel C Tolerance Level=0.05				
	Excess Return		Cumulative Return	
	Portfolio	Benchmark	Portfolio	Benchmark
Mean	0.0052	0.0024	1.1534	1.0752
Variance	0.0016	0.0003	0.0511	0.0073
Standard Deviation	0.0402	0.0173	0.2260	0.0853
Sharpe Ratio (I)	0.1050		0.4920	
Sharpe Ratio (II)	0.0854	0.0388	0.3345	-0.0304

regime can be calculated. Table 9 lists the portfolio performance statistics under regime-dependent strategy in the bull market. Compared to the overall portfolio performance, the portfolio performance for the bull market is similar but presents higher mean returns, volatilities and Sharpe ratios.

The only other regime in the study period was the “transition” market and the statistics for that implied regime are provided in Table 10. The advantage of the regime-dependent strategy is not so obvious in this case.

An interesting issue is the number of days in the study period that the regime-dependent strategy outperforms the benchmark strategy. This result is shown in Table 11.

In the overall experiment period, 52 out of 150 days are in the bull market. In the bull market, about 60% of the time the regime-dependent strategy out-performs the benchmark at each tolerance level. If cumulative returns are considered, where the

Table 9: Bull Market Performance

Tolerance Level=0.01				
	Return		Cumulative Return	
	Portfolio	Benchmark	Portfolio	Benchmark
Mean	0.00486	0.00359	1.10076	1.09853
Variance	0.00187	0.00059	0.01963	0.01083
Standard Deviation	0.04329	0.02424	0.14011	0.10409
Sharpe Ratio	0.057		0.053	
Tolerance Level=0.03				
	Excess Return		Cumulative Return	
	Portfolio	Benchmark	Portfolio	Benchmark
Mean	0.01060	0.00359	1.29575	1.09853
Variance	0.00397	0.00059	0.07990	0.01083
Standard Deviation	0.06301	0.02424	0.28267	0.10409
Sharpe Ratio	0.1732		1.0955	
Tolerance Level=0.05				
	Excess Return		Cumulative Return	
	Portfolio	Benchmark	Portfolio	Benchmark
Mean	0.01127	0.00359	1.32607	1.09853
Variance	0.00421	0.00059	0.08832	0.01083
Standard Deviation	0.06485	0.02424	0.29719	0.10409
Sharpe Ratio	0.181		1.169	

excess returns are carried forward, the percentage of days with out-performance is much higher, more in the order of 85%.

For the days in the transition market (98 days), the out-performance is about 50% of the time and this is consistent with the comparable statistics for both strategies in that regime.

Table 10: Transition Period Performance

Tolerance Level=0.01				
	Return		Cumulative Return	
	Portfolio	Benchmark	Portfolio	Benchmark
Mean	0.00236	0.00177	1.08492	1.06287
Variance	0.00015	0.00015	0.01025	0.00503
Standard Deviation	0.01242	0.01221	0.10122	0.07092
Sharpe Ratio	0.057		0.586	
Tolerance Level=0.03				
	Return		Cumulative Return	
	Portfolio	Benchmark	Portfolio	Benchmark
Mean	0.00207	0.00177	1.05797	1.06287
Variance	0.00022	0.00015	0.00960	0.00503
Standard Deviation	0.01489	0.01221	0.09800	0.07092
Sharpe Ratio	0.033		-0.148	
Tolerance Level=0.05				
	Return		Cumulative Return	
	Portfolio	Benchmark	Portfolio	Benchmark
Mean	0.00192	0.00177	1.06182	1.06287
Variance	0.00024	0.00015	0.00759	0.00503
Standard Deviation	0.01542	0.01221	0.08713	0.07092
Sharpe Ratio	0.015		-0.043	

Table 11: Days of Out-performance

Bull Regime (52 days)				
Tolerance Level	0.01	0.03	0.05	1
Daily return	30	31	30	31
Cumulative return	29	44	45	52

Transition Regime (98 days)				
Tolerance Level	0.01	0.03	0.05	1
Daily return	51	51	54	54
Cumulative return	67	44	51	51

6 Conclusion

This paper proposes and evaluates a regime-dependent market neutral strategy that maximizes alpha with limited exposures to style and macro risk factors. This strategy is implemented by solving an asset allocation optimization problem for the nine sector ETFs. The strategy is then compared to the benchmark strategy which invests passively and equally among the nine sector ETFs. In general, the regime-dependent strategy outperforms the benchmark strategy. The regime-dependent strategy appears to be much more attractive in the bull market than in the transition market, as its mean returns and Sharpe ratios are much higher in that market. In the transition market, the regime-dependent strategy also slightly outperforms the benchmark strategy. In our evaluation period, the bear market is not present.

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