

ASYMPTOTICALLY DISTRIBUTION-FREE STATISTICAL TEST FOR GENERALIZED LORENZ CURVES: AN ALTERNATIVE APPROACH

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A generalized Lorenz (GL) curve differs from a Lorenz curve in that the former is a rescaled version of the latter. A GL curve represents the relationship between the average income computed from a cumulative percentage of the population and the corresponding cumulative percentage. GL dominance is a useful criterion for ranking GL curves either for an economy over time or for a number of economies at one point in time. Relative to a dominated GL curve, a dominating GL curve indicates both that total income for the population is higher and that it is more equally distributed. Hence, it is obviously more desirable in a certain social sense. While sound statistical tests are essential for making statistical inference about GL dominance from sample GL curve estimates, the lack of a suitable joint test procedure for GL dominance is an unsolved problem in income distribution literature. This paper aims at solving this problem and provides an illustrative empirical example to show how to apply this test procedure in empirical research.

1. INTRODUCTION

A generalized Lorenz (GL) curve differs from a Lorenz curve in that the former is a rescaled version of the latter. As demonstrated in Figure 1, a GL curve (such as curves A and B) graphs the relationship between the average income computed for a cumulative percentage of the population and the corresponding cumulative percentage. A Lorenz curve (such as curves C and D) graphs the relationship between the cumulative percentage of income against the cumulative percentage of the population. Both begin with the lowest income group.

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Generalized Lorenz Curves

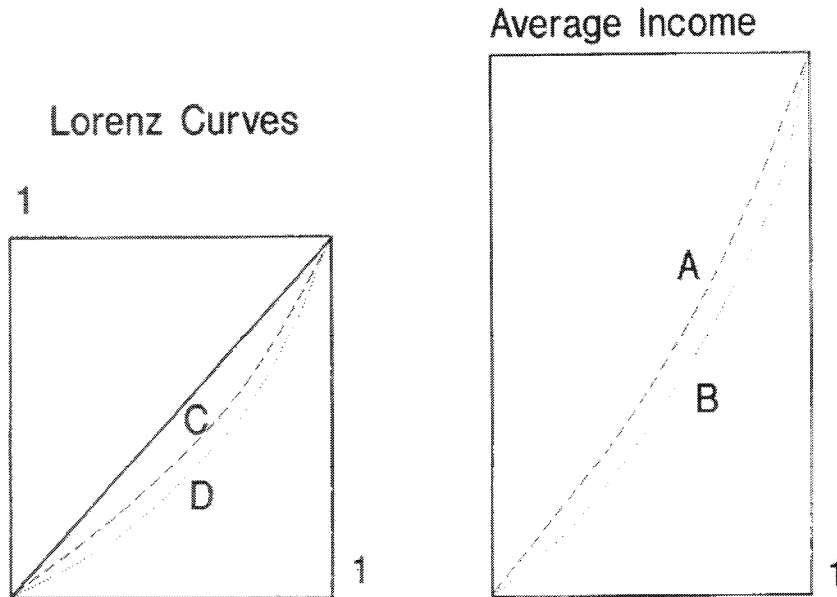


Figure 1. Lorenz Curves and Generalized Lorenz Curves

GL dominance is a useful criterion for ranking GL curves in social and economic research. This criterion is proposed by Shorrocks (1983) as an extension of Atkinson's (1970) Lorenz dominance criterion to allow for unequal means of income distributions. When GL curves are evaluated based on the GL dominance criterion, the evaluation is consistent with the social planner's preferences characterized by all monotonically increasing, anonymous, equality-preferring social welfare functions. In other words, relative to a dominated GL curve, a dominating GL curve indicates both that total income for the population is higher and that it is more equally distributed. Hence, it is obviously more desirable from the social planners' viewpoint.

Because GL dominance can reflect the aforementioned general characteristics of the social planner's preference, GL dominance analysis becomes a practical tool for evaluating income distributions either for an economy over time, or for a number of economies at one point in time, or for both. In addition, when the GL dominance criterion is used in assessing social welfare, there is no need to rely on any specific parametric social welfare function. Hence, GL dominance analysis is less restrictive, and increasingly used in applied research.

Traditionally, whether or not one GL curve dominates another is judged based on graphical examination. Recently, economists and other social scientists became aware of the drawback of such practice. As a sample mean computed from a spe-

cific sample, a set of GL curve estimates computed from a sample of incomes does provide useful information about its population counterpart, but it is still subject to sampling variation. Hence, statistical procedures are needed for making inferences which are more reliable than the ones based on graphical examination. This can be seen from a series of related studies. For example, statistical inference for Lorenz dominance has been developed by Beach and Davidson (1983), Gastwirth and Gail (1985), and Beach and Richmond (1985). Recently, hypothesis testing for Lorenz dominance has been discussed in Anderson (1996), Beach, Davidson, and Slotsve (1994), Bishop, Chakraborti, and Thistle (1989, 1994), Bishop, Formby, and Smith (1991), Bishop, Formby, and Thistle (1992), Xu (1994), and Xu, Fisher, and Willson (1994).

Because of increasing awareness of the usefulness of statistical procedures, Bishop et al. (1989) proposed a new distribution-free test procedure for GL dominance (briefly referred to as the BCT test). Essential elements of the BCT test procedure are two-stage, multiple pairwise sub-tests: first, to test for equality, two GL curves at preselected percentiles (or deciles) are estimated, and the equality of each and every pair of estimates of the two GL curves at a specific percentile (or decile) is tested; and, second, to test for GL dominance, multiple pairwise sub-tests for inequality are conducted, and these results are then combined to draw a conclusion on GL dominance between the two GL curves being compared.

Bishop et al. (1989) made two contributions to the literature: first, they developed the procedure of statistical inference for GL dominance which was not dealt with before; second, they highlighted the difficulty and importance of hypothesis testing for a dominance relation. However, the BCT test procedure suffers from some weaknesses. Subsequently, Bishop et al. (1991, 1992, 1994) modified their tests in various contexts while maintaining their two-stage testing strategy with the second stage being the union-intersection test which is based on multiple pairwise sub-tests.

The lack of a suitable joint statistical test procedure for GL dominance is an unsolved problem in income distribution literature. This paper offers an alternative test procedure based on Beach and Davidson (1983), Kodde and Palm (1986), Wolak (1989a, 1989b), Xu (1994), and Xu et al. (1994). This new alternative combines the two-stage, multiple sub-tests into one single test procedure and, hence, is conceptually much simpler. This alternative test is particularly useful if one wants to test for GL dominance directly instead of testing GL dominance through the two-stage, multiple sub-tests.

The basic idea of the new test procedure, as given by equation (4) in Theorem 1, is much simpler than it appears to be. When one compares two GL curves at various percentiles (or deciles), one can compute the differences between two sets of GL curve estimates in two different ways: one is to estimate the *unrestricted* differences, i.e., "let the data speak for themselves"; and the other is to estimate the *restricted* differences, i.e., "let the null hypothesis of GL dominance speak for itself." The former reflects the information carried by the data, while the latter is a

result of the null hypothesis. If the null hypothesis is true in the sense that the data are generated as specified by the null hypothesis, then the two sets of differences, *unrestricted* and *restricted*, would be the same or statistically close to each other. Otherwise, the two sets of differences would be far apart statistically. The test-statistic developed in this paper is a measure that can gauge the gap between the two sets of differences statistically. The value of the test-statistic will be low (high) if the gap is small (large). Since the test-statistic has a fairly complex distribution, i.e., it behaves as a weighted sum of χ^2 random variables with different degrees of freedom, critical values have to be determined in a unique way as explained in the later part of the paper.

This paper also provides an application of the proposed test procedure to Canadian wage distributions in the period of 1986–1990. This work is of independent interest, because the empirical results reveal some scientific evidence which could not be observed by using some conventional summary or casual observation. For example, the results show that when GL dominance is the criterion, the wage distribution improved in 1987 compared to 1986. Improvement was less pronounced in the middle of the period. At the end of the period, the wage distribution only experienced some uneven and minor changes. To economists and other social scientists, this indicates that the wage distribution in the late 1990s did not experience significant improvement in terms of increased levels and decreased inequality. This information characterizes the labor market condition in the late 1980s when more part-time jobs have replaced some highly paid, more secure jobs. Social welfare hence was also affected.

The remainder of the paper is organized as follows. In Section 2, based on the evaluation of existing GL dominance tests, an alternative test is proposed and discussed. In Section 3, an application of the proposed test procedure to the Canadian wage distributions is presented as an example. Finally, some concluding remarks are offered in Section 4.

2. GENERALIZED LORENZ DOMINANCE: AN ALTERNATIVE TEST PROCEDURE

Let F denote the population distribution function for incomes; and μ denote the mean of incomes. Let $Y(p) \equiv \{y : F(y) \geq p\}$ be the quantile function. The ordinary Lorenz curve is defined as $L(p) \equiv (1/\mu) \int_0^p Y(u) du$. The GL curve is given by $G(p) \equiv \mu L(p) = \int_0^p Y(u) du$, where $G(0)=0$ and $G(1)=\mu$ (see Shorrocks, 1983). In income distribution literature, it is conventional to evaluate GL curves at K points. These K points are chosen such that $0 < p_1 < p_2 < \dots < p_K = 1$ and $p_i = i/K$ with $i = 1, 2, \dots, K$. The K population quantiles corresponding to these p_i 's are denoted by $\xi_i = Y(p_i)$, $i = 1, 2, \dots, K$. The conditional mean and variance of income less than ξ_i are $\gamma_i = E(Y | Y \leq \xi_i)$ and $\lambda_i^2 = E[(Y - \gamma_i)^2 | Y \leq \xi_i]$, respectively, for $i = 1, 2, \dots, K$, and γ_K and

λ_K^2 are the overall or unconditional mean and variance, respectively. The vector of GL ordinates at p_1, p_2, \dots, p_K is given by $\theta = [p_1\gamma_1, p_2\gamma_2, \dots, p_K\gamma_K]'$; (see Beach and Davidson, 1983). The population quantities can be estimated consistently by their sample analogs which are denoted by the same symbols with hats.

The BCT test procedure is designed to test for a GL dominance relation between two GL curves, say a and b , the ordinates of which are denoted by θ^a and θ^b , respectively. The test is based on the following lemma.

Lemma 1: *If the distribution function is strictly monotonic, twice differentiable and has finite mean and variance, and the estimated vector of GL ordinates at p_1, p_2, \dots, p_K is given by $\hat{\theta} = [p_1 \hat{\gamma}_1, p_2 \hat{\gamma}_2, \dots, p_K \hat{\gamma}_K]'$, then*

$$\sqrt{N}(\hat{\theta} - \theta) \rightarrow \mathcal{N}(0, \Sigma),$$

where, for $i \leq j$, the ij -th element of Σ is given by

$$\sigma_{ij} = p_i [\lambda_i^2 + (1 - p_j)(\xi_i - \gamma_i)(\xi_j - \gamma_j) + (\xi_i - \gamma_i)(\gamma_j - \gamma_i)].$$

Proof. See Beach and Davidson (1983). \square

Assuming that the two samples are independent and H_0^1 : $\theta^a = \theta^b$, $\hat{\theta}^a - \hat{\theta}^b$ is asymptotically distributed as $\mathcal{N}(0, \Omega)$ where $\Omega = \Sigma^a / N^a + \Sigma^b / N^b$. Bishop et al. (1989) suggest that a test for the equality of two GL curves (H_0^1 : $\theta^a = \theta^b$ against H_a^1 : $\theta^a \neq \theta^b$) could be based on the test-statistic:

$$T_1 = (\hat{\theta}^a - \hat{\theta}^b)' \hat{\Omega}^{-1} (\hat{\theta}^a - \hat{\theta}^b), \quad (1)$$

which is asymptotically distributed as $\chi^2(K)$ under the null hypothesis H_0^1 . In addition, they suggest the statistic T_2 could be used to test for GL dominance, viz., H_0^2 : $\theta^a = \theta^b$ against H_a^2 : $\theta^a > \theta^b$, the test-statistic T_2 should be employed.

$$T_2 = \mathbf{1}'_K (\hat{\theta}^a - \hat{\theta}^b) / (\mathbf{1}'_K \hat{\Omega}^{-1} \mathbf{1}_K)^{1/2} \quad (2)$$

is asymptotically distributed as $\mathcal{N}(0, 1)$ under the null hypothesis H_0^2 . Note $\mathbf{1}_K$ is a column vector of K ones.

While the statistic T_1 is a standard χ^2 test-statistic, the test-statistic T_2 is somewhat unique. T_2 is intended to be useful for testing a dominance relation. Because of the nature of dominance relations, using T_1 itself would not constitute a proper test for GL dominance. As Bishop et al. (1989) correctly point out: "If the question is simply whether $F^a = F^b$ or not, then other nonparametric tests are available, the K-S test being the best known. However, for the test of H_0^2 : $\theta^a \leq \theta^b$ against

$H_a^2: \theta^a > \theta^b$, the alternative is second degree stochastic dominance. There is no other test for second degree stochastic dominance." Thus, the test-statistic T_2 was proposed to circumvent this problem. Unfortunately, T_2 is unable to test for the distance between two GL curves. More specifically, the numerator of T_2 , $\mathbf{1}'_K (\hat{\theta}^a - \hat{\theta}^b)$, is the sum of the point-wise distance measures and the denominator $(\mathbf{1}'_K \hat{\Omega}^{-1} \mathbf{1}_K)^{1/2}$ normalizes this sum. Thus, T_2 is a valid statistic for testing the null hypothesis $H_0^2: \mathbf{1}'_K (\theta^a - \theta^b) \leq 0$ against the alternative hypothesis $H_a^2: \mathbf{1}'_K (\theta^a - \theta^b) > 0$, but not for the null hypothesis $H_0^2: \theta^a \leq \theta^b$ against the alternative hypothesis $H_a^2: \theta^a > \theta^b$.

As a remedy, Bishop et al. (1992) adopt a point-wise testing strategy following the idea due to Beach and Richmond (1985). In this context, the null subhypotheses and corresponding alternative subhypotheses are $H_{0,i}^2: \theta_i^a = \theta_i^b$, against $H_{a,i}^2: \theta_i^a \neq \theta_i^b, i = 1, 2, \dots, K$, where $\theta_i^a(\theta_i^b)$ is the i -th element in $\theta^a(\theta^b)$. The test-statistics $T_2^i, i = 1, 2, \dots, K$, are defined as

$$T_2^i = \frac{\hat{\theta}_i^a - \hat{\theta}_i^b}{[(\hat{\sigma}_{ii}^a/N^a) + (\hat{\sigma}_{ii}^b/N^b)]^{1/2}}, \quad (3)$$

where $\hat{\sigma}_{ii}$ is the i -th diagonal element of $\hat{\Sigma}$, for $i = 1, 2, \dots, K$. The subhypotheses must be tested simultaneously, holding the probability of rejecting the *overall* null hypothesis ($H_0^2: \theta^a = \theta^b$) fixed. The critical values for these tests are obtained from the distribution of the Studentized maximum modulus (SMM) variate (See Miller, 1981, for a discussion of the SMM distribution, and Stoline and Ury, 1979, for the statistic tables). The dominance relation can be inferred as follows: Dominance results if there is at least one positive significant difference and no negative significant differences between GL ordinates. Two GL curves are ranked as equivalent if there are no significant differences. The curves cross if the difference in at least one set of differences are positive and significant while at least one other set are negative and significant.

There are two issues worth discussing. The first issue is about outcome partition in theoretical dominance analysis. There are four possible outcomes from theoretical dominance analysis of two GL curves. These are:

- (1) θ^a and θ^b are identical;
- (2) θ^a strictly dominates θ^b ;
- (3) θ^b strictly dominates θ^a ; and
- (4) Neither θ^a nor θ^b strictly dominates the other.

It is also possible to change the outcome partition. Using the dominance criterion in weak form, there are three possible outcomes: (1') θ^a dominates θ^b ; (2') θ^b dominates θ^a ; and (3') neither θ^a nor θ^b dominates the other. This outcome partition

does not change things significantly. Because cases (1') and (2') imply case (1). That case (1') not case (2') [case (2') not case (1')] holds implies that case (2) [case (3)] holds. The purpose of dominance testing is to find if a dominance relation exists in statistical sense. It is known, however, that it is case (4) [or case (3')] that causes difficulty in dominance analysis.¹

The second issue is about econometric testing. To differentiate the four possible outcomes, it is often necessary to use a *two-step* test procedure with the second-step being *simultaneous* statistical sub-tests to differentiate these outcomes as described in the above. The second-step test based on simultaneous sub-tests is also called the union-intersection test. The question is: Is there any other alternative test procedure which may test a dominance relation in weak form in one-step without using the union-intersection test? The answer is "Yes." There is a line of literature in statistics and econometrics (see for example Kodde and Palm, 1986) which is relevant to the alternative test procedure but has not been given enough attention. In particular, this alternative procedure could circumvent the difficulty caused by case (4) [or (3')] as explained in the following.

The basic idea of the alternative test procedure is relatively simple. It is based on the alternative partition of three possible outcomes in theoretical dominance analysis. This procedure does not require testing K null subhypotheses against their corresponding alternative subhypotheses. What it does is to construct a test-statistic based on the unrestricted estimates of $\theta^a - \theta^b$ and corresponding restricted estimates of $\theta^a - \theta^b$ subject to the inequality constraint $\theta^a - \theta^b \geq 0$. The test-statistic, T_3 , is a normalized measure between the two sets of estimates. If the two sets of estimates coincide thus indicating the the inequality constraint is not binding, the value of the test-statistic will approach zero and the null hypothesis of $\theta^a - \theta^b \geq 0$ will not be rejected. If the unrestricted estimates are located in the unrestricted parameter space while the restricted estimates are located in the restricted parameter subspace, the value of the test-statistic will deviate from zero, and the null hypothesis will be rejected if the value of the test-statistic is significantly from zero. This alternative test procedure has three advantages. First, a dominance relation, such as case (1') [case (2')], is explicitly specified under the null hypothesis, and other outcomes, such as cases (2') and (3') [cases (1') and (3')], are automatically excluded from the null hypothesis and included in the alternative hypothesis. Second, the test is straightforward because there is no need for simultaneous point-wise sub-tests. Third, even if a researcher is interested in testing for strict dominance, he or she could simply use the same test procedure to test cases (1') and (2'), respectively. The possible conclusions from the two tests are: θ^a and θ^b are identical, θ^a strictly dominates θ^b , or θ^b strictly dominates θ^a , as shown in our empirical example.

The new test procedure specifies explicitly a GL dominance relation under the null hypothesis and no such GL dominance relation under the alternative hypothesis. The test procedure is based on the minimum distance principle.

Let $\hat{\Theta} = [\hat{\theta}^a \hat{\theta}^b]'$, $\Theta = [\theta^a \theta^b]'$ and $\Lambda = \begin{bmatrix} \Sigma^a & 0 \\ 0 & \Sigma^b \end{bmatrix}$. The following lemma provides a framework for the new test procedure.

Lemma 2: *If a $2K \times 1$ vector Θ can be consistently estimated by $\hat{\Theta}$ based on two samples of size N such that $\sqrt{N}(\hat{\Theta} - \Theta) \rightarrow \mathcal{N}(0, \Lambda)$, then for testing $H_0: h(\Theta) \geq 0$ against $H_a: h(\Theta) \not\geq 0$ with $h: \mathcal{R}^{2K} \rightarrow \mathcal{R}^K$, the test-statistic D is defined as $D = \|\hat{\psi} - \tilde{\psi}\|_{\Xi} = (\hat{\psi} - \tilde{\psi})' \Xi^{-1} (\hat{\psi} - \tilde{\psi})$, where $\psi = \sqrt{N}h(\Theta)$, $\hat{\psi} = \sqrt{N}h(\hat{\Theta})$, and $\tilde{\psi} = \sqrt{N}h(\tilde{\Theta})$. $\hat{\psi}$ is an unrestricted estimator and has large sample variance-covariance matrix $\Xi = (\partial h / \partial \Theta)' \Lambda (\partial h / \partial \Theta)$. $\tilde{\psi}$ is a restricted estimator solving $\min_{\psi} (\hat{\psi} - \psi)' \Xi^{-1} (\hat{\psi} - \psi)$ subject to the constraint $\psi \geq 0$. D has a large sample distribution*

$$\sup_{\psi \geq 0} Pr(D \geq q | \Xi) = \sum_{i=0}^K Pr[\chi^2(K-i) \geq q] W(K, i, \Xi)$$

with W denoting the probability that i of the K elements of $\tilde{\psi}$ are strictly positive, and q denoting the critical value.

Proof. See Kodde and Palm (1986), Xu (1994), and Xu, Fisher and Willson (1994). \square

Remarks: (1) Λ is given a simpler structure here for simplicity and relevancy. Lemma 2 allows for more complex structure (see Xu, 1994; Xu, Fisher, and Willson, 1994). (2) The upper- and lower-bounds of critical values for testing inequality restrictions are provided by Kodde and Palm (1986).² Decision rules based on the statistic D are: if D exceeds the upper-bound value, reject H_0 ; and if D is smaller than the lower-bound value, do not reject H_0 . If D is in the inconclusive region, then the weights W in the distribution can be determined numerically, and D can be compared with the critical value corresponding to the chosen significance level α .

Based on Lemma 2, the new test-statistic T_3 is given in the following main theorem. Define a $(K \times 2K)$ matrix $H \equiv [I_K \ : \ -I_K]$ so that $H\hat{\Theta} = \hat{\theta}^a - \hat{\theta}^b$ and $\frac{1}{N}H\hat{\Lambda}H' = (\hat{\Sigma}^a/N + \hat{\Sigma}^b/N)$.

Theorem 1: *To test $H_0^3: H\Theta = \theta^a - \theta^b \geq 0$ against $H_a^3: H\Theta = \theta^a - \theta^b \not\geq 0$, the test-statistic T_3 for GL dominance is given by:*

$$T_3 = \Delta' \left[\frac{1}{N} H \hat{\Lambda} H' \right]^{-1} \Delta, \quad (4)$$

where $\Delta = [(\hat{\theta}^a - \hat{\theta}^b) - (\bar{\theta}^a - \bar{\theta}^b)]$; $\hat{\theta}^a$, $\hat{\theta}^b$, and $\hat{\Lambda}$ are the unrestricted estimates while $\bar{\theta}^a$ and $\bar{\theta}^b$ are the restricted estimates minimizing

$$[(\hat{\theta}^a - \hat{\theta}^b) - (\theta^a - \theta^b)]' \left[\frac{1}{N} H \hat{\Lambda} H' \right]^{-1} [(\hat{\theta}^a - \hat{\theta}^b) - (\theta^a - \theta^b)] \quad (5)$$

$$s.t. (\theta^a - \theta^b) \geq 0$$

The test-statistic T_3 is asymptotically distributed as a weighted sum of χ^2 random variables with different degrees of freedom:

$$\sup_{\Theta \geq 0} Pr \left(T_3 \geq q \mid \frac{1}{N} H \hat{\Lambda} H' \right) = \sum_{i=0}^K Pr[\chi^2(K-i) \geq q] W \left(K, i, \frac{1}{N} H \hat{\Lambda} H' \right)$$

The decision rules based on the statistic T_3 are the same as those for the statistic D in Lemma 2.

Proof: This result is a consequence of Lemma 2. \square

Remarks: (1) The alternative hypothesis $H_a^3 : \theta^a - \theta^b \geq 0$ given in Theorem 1 is not equivalent to $\theta^a - \theta^b < 0$. The former is weaker than the latter. While $\theta^a - \theta^b \geq 0$ represents a weak form of GL dominance, $\theta^a - \theta^b \geq 0$ represents any violation of $\theta^a - \theta^b \geq 0$. (2) The variance-covariance matrix $\frac{1}{N} H \hat{\Lambda} H' = (\hat{\Sigma}^a/N + \hat{\Sigma}^b/N)$ in

Theorem 1 should be replaced by $(\hat{\Sigma}^a/N^a + \hat{\Sigma}^b/N^b)$ when the two sample sizes are large but not equal.

The advantage of the proposed test-statistic T_3 over the BCT test-statistics T_1 , T_2 , and T_2^i , $i = 1, 2, \dots, K$, is that T_3 can be used to test a dominance relation directly. This proposed test procedure is able to measure effectively the deviation from a dominance relation. When one income distribution dominates the other, the test-statistic T_3 approaches zero; otherwise, T_3 will have a positive value. The finite sample properties of the test procedure have been evaluated using the Monte Carlo method (Xu, 1994). It is found that the test has desirable properties.

3. ILLUSTRATIVE EXAMPLE: CANADIAN WAGE DISTRIBUTIONS, 1986—1990

It is well recognized that during the 1980s transfer payments (such as social security and unemployment insurance benefits) played an important role in alleviating

Table 1
The Generalized Lorenz Ordinates, 1986–1990

Decile	1986	1987	1988	1989	1990
0.1	99.38 (1.67)	113.68 (1.67)	113.46 (1.77)	119.18 (1.83)	118.94 (1.90)
0.2	403.09 (4.85)	455.80 (5.08)	447.82 (5.25)	459.72 (5.35)	469.86 (5.65)
0.3	965.33 (10.00)	1087.31 (10.37)	1053.50 (10.75)	1072.20 (10.80)	1096.30 (11.17)
0.4	1851.34 (16.43)	2057.51 (16.57)	1992.60 (17.38)	2009.60 (17.21)	2043.30 (17.53)
0.5	3093.92 (24.16)	3385.53 (23.03)	3296.00 (24.86)	3290.20 (24.36)	3331.80 (24.74)
0.6	4730.01 (31.44)	5104.47 (30.04)	4987.80 (32.32)	4953.60 (31.69)	4999.10 (32.04)
0.7	6763.83 (38.56)	7218.58 (36.40)	7083.80 (39.53)	7014.30 (39.21)	7065.10 (39.43)
0.8	9273.23 (46.21)	9806.27 (43.66)	9670.50 (47.89)	9553.10 (46.91)	9599.50 (47.29)
0.9	12402.12 (54.02)	13001.72 (50.67)	12911.00 (56.24)	12730.00 (55.35)	12765.00 (55.62)
1.0	17002.00 (67.12)	17737.58 (65.10)	17714.00 (70.42)	17521.00 (70.76)	17461.00 (69.56)

Notes: Wage measured in 1986 Canadian dollars. Estimated standard errors in parentheses.

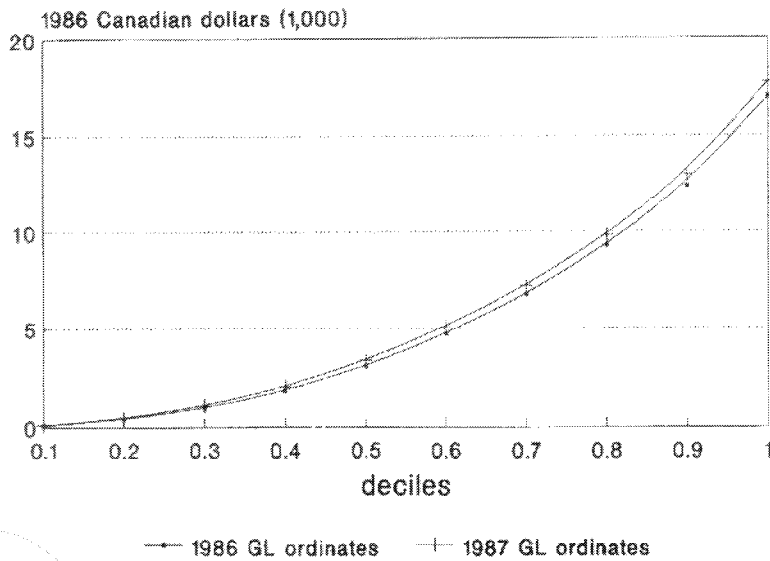


Figure 2. The Generalized Lorenz Ordinates: 1986 vs 1987

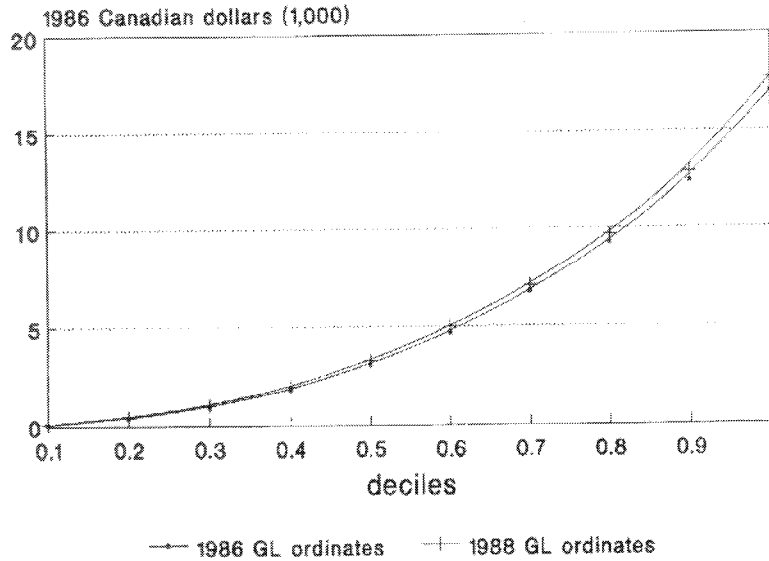


Figure 3. The Generalized Lorenz Ordinates: 1986 vs 1988

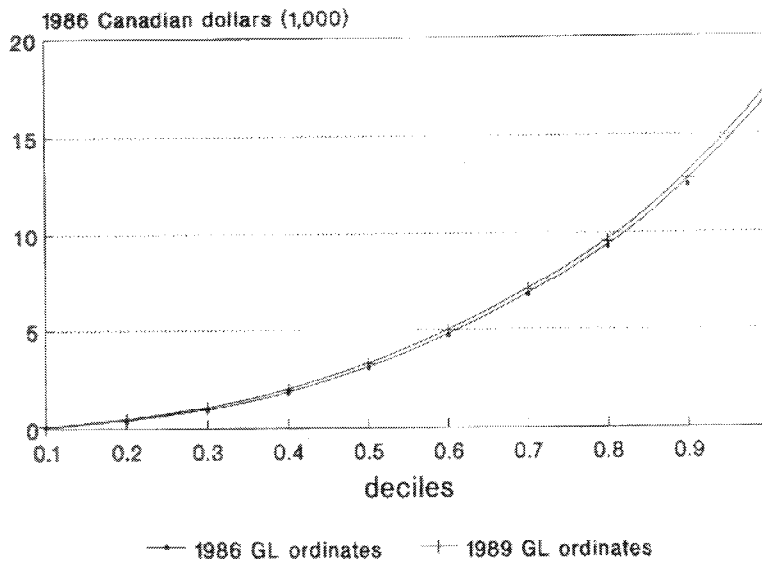


Figure 4. The Generalized Lorenz Ordinates: 1986 vs 1989

income inequality in Canada (see Osberg, Erksøy, and Phipps, 1998). Thus, to examine labor income inequality, it is often necessary to examine wage distributions which are not distorted by transfer payments. When the issue of wage distributions is addressed, GL curves and dominance become relevant, useful analytical tools. This is because GL curves reveal changes in both level and dispersion of

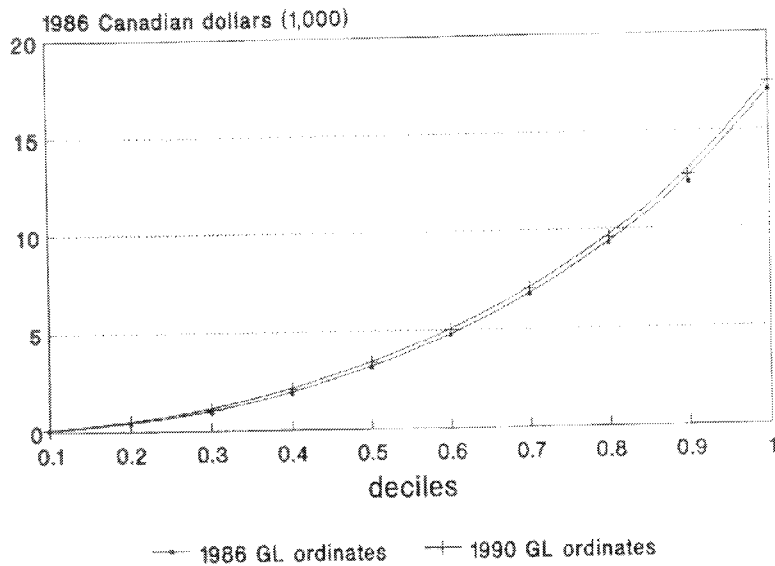


Figure 5. The Generalized Lorenz Ordinates: 1986 vs 1990

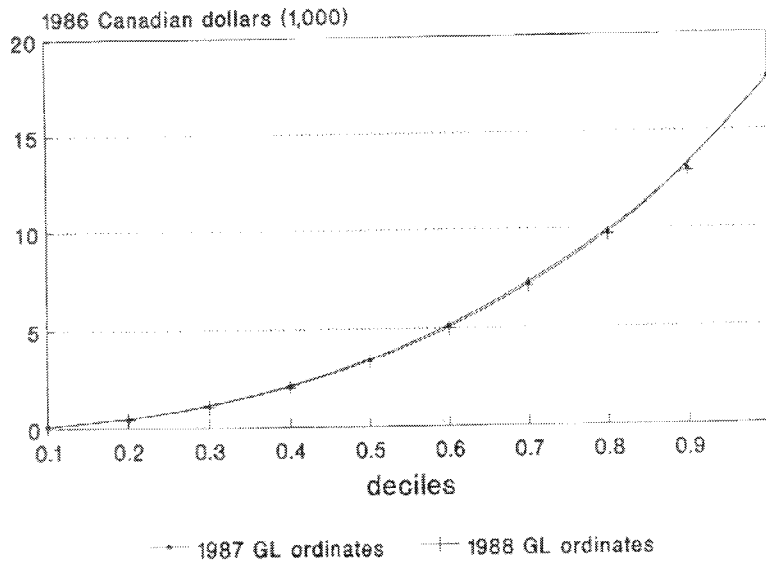


Figure 6. The Generalized Lorenz Ordinates: 1987 vs 1988

wage distributions and the GL dominance criterion is useful for ranking wage distributions over time based on the generally acceptable social welfare criteria.

The Canadian Labour Market Activity Survey (LMAS) conducted by Statistics Canada during the period of 1986-1990 provides a unique, large survey data source for evaluating Canadian wage distributions. The sample sizes of the survey varied over the years. The sample size for 1986 is 66,934. It changes to 77,802 in

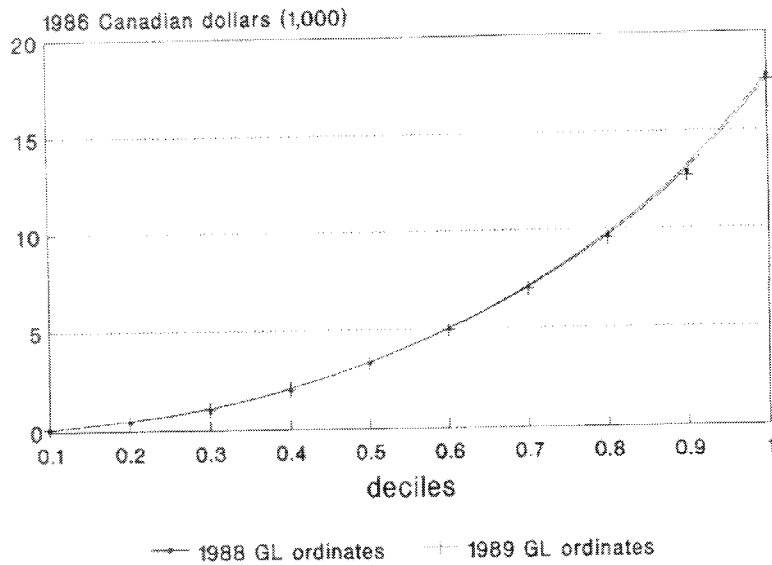


Figure 7. The Generalized Lorenz Ordinates: 1988 vs 1989

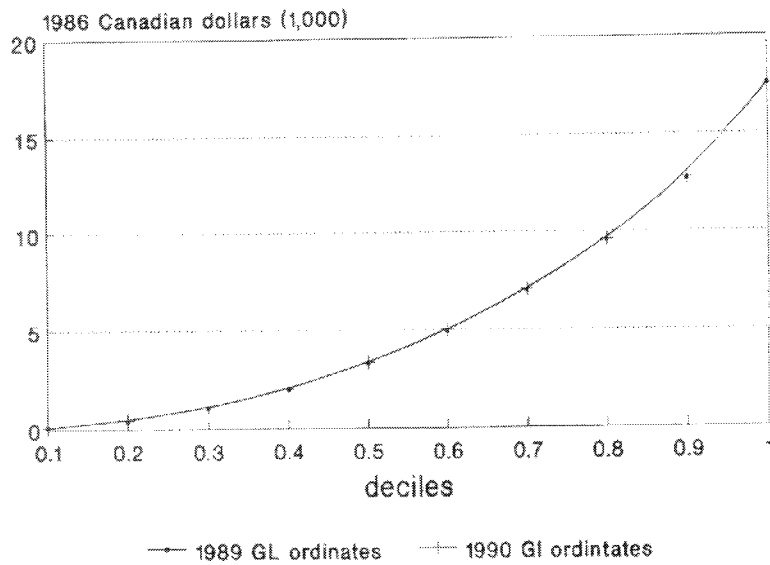


Figure 8. The Generalized Lorenz Ordinates: 1988 vs 1989

the 1987 survey. The sample sizes for the 1989 and 1990 surveys are 63,804 and 63,660, respectively. The annual earnings for all jobs are computed from hourly wage rates and hours worked in the corresponding calendar year.³ The nominal wages are then transformed into the real wages in 1986 Canadian dollars.

Table 1 provides the estimated GL curve ordinates and corresponding standard errors (in parentheses) based on the LMAS of 1986—1990. These generalized

Table 2
Test Results Based on the 1986 GL Curve Ordinates

Testable Hypothesis under H_0	T_3
GL_{87} Dominates GL_{86}	0.000
GL_{86} Dominates GL_{87}	86.524
GL_{88} Dominates GL_{86}	0.000
GL_{86} Dominates GL_{88}	66.446
GL_{89} Dominates GL_{86}	0.000
GL_{86} Dominates GL_{89}	72.813
GL_{90} Dominates GL_{86}	0.000
GL_{86} Dominates GL_{90}	81.832

Notes: GL_{xx} represents the GL curve ordinates for year XX. The lower- and upper-bounds of the critical value at the 5 percent significance level are 2.706 and 17.670, respectively. If a test-statistic is less than 2.706, do not reject the null hypothesis; if a test-statistic is greater than 17.670, reject the null hypothesis. The results in this table show that the 1986 GL curve ordinates are dominated by the 1987, 1988, 1989, and 1990 GL curve ordinates, respectively.

Lorenz ordinates are given in Figures 2-8. When the estimated GL curve ordinates are compared over time, the following observations are made. First, the 1987, 1988, 1989, and 1990 GL curve ordinates are higher than their 1986 counterparts at all deciles. Second, the 1988 GL curve ordinates are lower than their 1987 counterparts at all deciles. Third, the 1988 and 1989 GL curve ordinates cross once, and the 1989 and 1990 GL curve ordinates cross twice. In other words, Canadian wage distributions have improved significantly from 1986 to 1987 in the sense of GL dominance; i.e., the wage distribution was improved showing higher wage levels and smaller wage inequality. A similar change did not occur since then. The GL curve ordinates decreased slightly at all deciles in 1988. From 1988 to 1989, the GL curve ordinates increased at the lower deciles (0.1-0.4) and decreased at the higher deciles (0.5-1.0). From 1989 to 1990, the GL curve ordinates increased at the middle-range deciles (0.2-0.9) while they decreased at two ends of the GL curve ordinates (0.1 and 1.0).

To verify the above observations, the new test procedure proposed above is employed. One of the nice characteristics of the new procedure is that although it is suitable for testing weak dominance, it is also useful for testing strict dominance. If a strictly dominates b , then it must be the case that a weakly dominates b and that b does not weakly dominate a . With this idea in mind, the results on testing for GL dominance in two directions are reported.

The test results reported in Table 2 confirm that the 1987-1990 GL curve ordinates dominate their 1986 counterparts. All the test-statistics for the 1987, 1988, 1989, and 1990 GL curve ordinates dominating the 1986 GL curve ordinates are zero and below the lower-bound of the critical value at the 5% significance level

Table 3
Test Results for 1987 and 1988 GL Curve Ordinates

Testable Hypothesis under H_0	T_3
GL_{87} Dominates GL_{88}	0.000
GL_{88} Dominates GL_{87}	7.432

Notes: GL_{XX} represents the GL curve ordinates for year XX. The lower- and upper-bounds of the critical value at the 5 percent significance level are 2.706 and 17.670, respectively. If a test-statistic is less than 2.706, do not reject the null hypothesis; if a test-statistic is greater than 17.670, reject the null hypothesis. The results in this table show that the 1987 GL curve ordinates dominate the 1988 GL curve ordinates.

Table 4
Test Results for 1988, 1989, and 1990 GL Curve Ordinates

Testable Hypothesis under H_0	T_3
GL_{88} Dominates GL_{89}	5.039
GL_{89} Dominates GL_{88}	5.254
GL_{89} Dominates GL_{90}	2.403
GL_{90} Dominates GL_{89}	0.362

Notes: GL_{XX} represents the GL curve ordinates for year XX. The lower- and upper-bounds of the critical value at the 5 percent significance level are 2.706 and 17.670, respectively. If a test-statistic is less than 2.706, do not reject the null hypothesis; if a test-statistic is greater than 17.670, reject the null hypothesis. The results in this table show that neither the 1987 GL curve ordinates nor the 1988 GL curve ordinates dominate the other. The 1989 and 1990 GL curve ordinates weakly dominate each other.

(2.706). When the test is conducted for the same pair of GL curve ordinates in the reverse direction (viz. the 1986 GL curve ordinates dominating the other GL curve ordinates), the test-statistics range from 66.446 to 72.813, and are much higher than the upper-bound of the critical value at the 5% significance level (17.670). The statistical evidence corroborate the observations based on Table 1.

Table 1 also suggests that the 1988 GL curve ordinates are lower than their 1987 counterparts. The test results in Table 3 appear to support this observation. While there is strong evidence that the 1987 GL curve ordinates dominate their 1988 counterparts (the test-statistic is zero), the reverse appears to have little statistical support (the test-statistic is 7.32).

The relations between the 1988 and 1989 GL curve ordinates, and between the 1989 and 1990 GL curve ordinates fall into a *gray* area, which is often characterized as a "neither-nor" case. From the test results in Table 4, it appears that neither the 1988 GL curve nor the 1989 GL curve dominates the other because both values

of the test-statistic are higher than the lower-bound of the critical value at the 5% significance level. It appears that the 1989 and 1990 GL curve ordinates might have been generated from the same data generating process because the values of the test-statistic are lower than the lower-bound of the critical value at the 5% significance level. For economists and other social scientists, the dynamics of Canadian GL curves demonstrates that the wage distribution was improved from 1986 to 1987. It was worsened in 1988. There was no significant change from 1989 to 1990.

4. CONCLUDING REMARKS

This paper has surveyed the existing tests for GL dominance and shown that the lack of a suitable joint test for GL dominance is an unsolved problem in income distribution literature. An alternative test procedure is proposed to avoid the weaknesses of these existing tests. The test-statistic T_3 and its asymptotic distribution are derived and discussed in detail. This paper demonstrates that the alternative test procedure is useful and simple to apply, and is an addition to the existing literature on GL dominance testing.

To demonstrate the application of this new test procedure, this paper reports the analysis of the Canadian wage distributions from 1986 to 1990 based on the GL dominance criterion. The test results confirm that the wage distribution had significantly improved in the sense of GL dominance from 1986 to 1987. Improvement was less pronounced since then. The 1988 GL ordinates worsened slightly from the 1987 counterparts. The wage distribution experienced uneven and minor changes in 1989 and 1990. This might reflect the fact that the improvement of the wage distribution was short-lived, and it became less equal and somewhat stagnated in real terms later during the survey period.

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NOTES

1. Beach, Davidson, and Slotsve (1994) attempt to solve a similar problem of crossing Lorenz curves based on Davies and Hoy (1995).
2. The reason for computing the upper- and lower-bonds for the critical value is that computing the weights W can be nontrivial. The computation of weights involves evaluation of K -multiple integrals, and closed forms are only available for a small K . Kodde and Palm (1986) provide a partial solution to this problem by computing the upper- and lower-bound critical values that do not require

computation of the weights. These bounds are given by $\alpha_l = \frac{1}{2} Pr(\chi_l^2 \geq q_l)$, and $\alpha_u = \frac{1}{2} Pr(\chi_{K-1}^2 \geq q_u) + \frac{1}{2} Pr(\chi_K^2 \geq q_u)$, where q_l and q_u are the lower- and upper-bounds, respectively, for the critical values of the test-statistic. A lower-bound for a critical value is obtained by choosing a significance level α and setting degrees of freedom (df) equal to one. An upper-bound for the critical value is obtained by choosing a significance level α and setting df equal to K .

3. When an individual did not work during the calendar year, the record is zero. Since wage distributions are evaluated for wage earners, these zero-record cases are eliminated.

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