Lower Inflation with Higher Volatility: What can be Learned from Recent Canadian Disinflation?

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Abstract

Whether the level of inflation and its variability are positively correlated is not only an issue of theoretical significance, but also an issue of policy importance. Although it has long been believed that there exists a positive relationship, and although the perceived relationship has sometimes been part of the framework within which the monetary policy choices are justified, this relationship has not been suitably studied for policy analysis. In this paper we examine the relationship between the level of inflation and its volatility using Canadian data from 1975 to 1995. We find no positive correlation between the level of inflation and its volatility during this period. Instead, we find a U-shape relationship. Despite the significant decline in the level of inflation after the Bank of Canada's implementation of a price stability policy in 1988, there is little evidence indicating a significant decline in the volatility of inflation. Our estimates also suggest that the level of inflation that minimizes the volatility of inflation is in the range of 2.2 to 4.5 percent per year.

Keywords: Inflation volatility, disinflation, Canada.

JEL Classification Number: E31, E52, E58.

1 Introduction

Among economists different views about the acceptable level of inflation have long been difficult to be reconciled. For instance, Okun (1971) once characterized those who prefer a lower inflation rate as "brake riders," and those who prefer a higher inflation rate as "gas pumpers." Since the adoption of price level stability as a policy goal by some monetary authorities, the differences have resurfaced in academic and public policy discussions. The recent debate in Canada is a case in point. While some advocate the zero inflation policy [see, e.g., Howitt (1990), and Laidler and Robson (1993)], others argue that the zero inflation policy has been too costly for the Canadian economy [see, e.g., Fortin (1996)]. However, despite the prevalent disagreement on the acceptable level of inflation, there is no disagreement among economists on the desirability of a stable level of inflation.

Many economists have pointed out the observed positive association between inflation rates and inflation volatility, and have argued that a high inflation environment necessitates more frequent changes in monetary policy. These observations and judgements have resulted in the belief that high inflation leads to increased inflation volatility [see, e.g., Okun (1971)]. Since inflation volatility appears to induce high costs of adjustment and to impede inflation forecasts, stabilizing the inflation rate at low levels so as to achieve a low inflation volatility seems to be an optimal policy choice. For example, Friedman (1977) argues that increased volatility of inflation leads to large welfare losses and inefficiencies in an economy until individuals and institutions adjust their expectations to high and variable inflation rates. According to Friedman, the costs of inflation variability are much larger than the costs associated with high rates of inflation, and much of the welfare gains associated with reducing inflation levels come from the resulting reduction in inflation variability. Therefore, a "natural" choice for stable inflation appears to be zero.

Is it the case that a monetary policy that aims at zero inflation minimizes inflation volatility? In this paper, we examine this question based on the inflation experience in Canada from 1975 to 1995. During the 1970s and 1980s Canada had relatively high inflation rates. In 1988, the Bank of Canada announced its new policy of price stability in an attempt to curb high inflation.[1]

[1]This policy change was marked by the Hanson Lecture delivered by the Governor of the Bank of

It is important to note that, while adopting the inflation targeting, the Bank of Canada emphasized the need for maintaining a stable price *level* as a way to reduce the volatility of inflation rate. As such, the zero inflation targeting seems to be justified on the ground that there exists a positive monotonic relationship between the inflation rate and its volatility, and that zero inflation policies eliminate policy-induced volatility.

The effect of monetary policy changes on inflation volatility are what matter for the policy evaluation, but this issue has been neglected in the literature. Therefore, in this paper, we attempt to establish an empirical association between inflation and its volatility with particular emphasis on the effects of monetary policy. We infer policy-induced changes in volatility from examining the correlations between some indicators of the monetary policy intentions and the volatility of inflation. This study also differs from earlier cross-country or country-specific studies on inflation volatility, because our data cover both high and low inflation periods.

We follow a rather unconventional route to present our results. First, we document our empirical findings, and then advance several theoretical arguments that may account for the association observed in the data. Since the relationship between the level of inflation and its volatility cannot be properly examined without a suitable measure of volatility, in section 2 we briefly review the existing literature. In the literature, inflation volatility has been measured in various ways, but not all of these measures are suitable for evaluating monetary policy changes. We focus on those volatility measures that are more appropriate for our analysis.

Our approach is intended to evaluate the link between inflation and its volatility during a relatively long period of time. We adopt a set of *local* volatility measures which are the centered moving-average variances and absolute deviations of the inflation rate over a certain time period or "window." Then, the relationship between the inflation rate and local volatility is modelled by both parametric and nonparametric regressions.

In section 3 we present and discuss our empirical results. The results show that the relationship between the inflation rate and its local volatility has a U-shape pattern, and that low inflation rates and high local volatility often coexist. The positive monotonic relationship exists only for a specific range of inflation rates, but the zero inflation rate

Canada at the University of Alberta on January 8, 1988. The text is reprinted in Crow (1988).

does not appear to correspond to the lowest local volatility of inflation. Our estimates suggest that the level of inflation that minimizes the volatility of inflation is in the range of 2.2 to 4.5 percent per year. To investigate the relationship under different monetary regimes, we split the sample into subperiods: before and after the implementation of the Bank of Canada's price stability policy in 1988. We find that the transition to low inflation coincides with *high* inflation volatility.

Some recent studies have focused on the link between the level of inflation and its uncertainty approximated by conditional standard deviations, and the most recent practice in measuring inflation uncertainty has been based on Kalman filter-ARCH models [see, e.g., Evans (1991)]. An ARCH approach may be useful to reveal how the uncertainty about the next period inflation rate is related to the current period inflation rate. However, the relationship between inflation and ARCH model-based inflationary uncertainty may not be very informative for the analysis of monetary policy changes. This is because uncertainty measures based on ARCH models are one-step-ahead conditional standard deviations. However, recent empirical research indicates that the impact of monetary policy changes typically takes months to fully materialize in the inflation process. Therefore, the temporal relationship between the one-step-ahead uncertainty and the current inflation rate is too short a time frame to be considered useful for policy analysis.

Nevertheless, for the purposes of comparison, we also specify and estimate a Kalman filter-ARCH model. Our estimates of inflation uncertainty do not provide any supporting evidence for ARCH effects. In addition, the correlations between the leads (and lags) of the inflation rate and the inflation uncertainty based on the Kalman filter-ARCH model are low and statistically insignificant.

In section 4 we discuss a number of theoretical considerations that may help explain some of our empirical findings. In particular, we examine whether high volatility of inflation during a transition from high to low inflation can be explained by (i) a rational expectations model in which the policymaker over-reacts to inflation, and (ii) policy credibility arguments. Section 5 concludes the paper.

2 Methodology

Inflation volatility (or variability), and uncertainty have been measured in various ways in the economics literature. However, these measures appear difficult to be reconciled within a single conceptual framework, and, as such, they often lead to conflicting conclusions. For example, as Khan (1977, p.824, footnote 19) pointed out: "...the conclusion about the role of variability depends on how the measure of variability is defined." How the measure of variability is defined, in turn, depends on the purpose. Therefore, we first briefly review of some of the existing measures, and then discuss which are suitable for the purposes of policy analysis.

2.1 Existing Measures

In the 1960s and 1970s, economists focused primarily on the link between the level of inflation and inflation volatility. Okun (1971) was one of the first who studied this relationship. The volatility measure used by Okun was essentially the standard deviation of the inflation rate over time and for a cross-section of economies. Based on a sample of 17 OECD countries covering the period from 1951 to 1968, he found a positive monotonic relationship between the mean inflation rates and their volatility.[2] Given that there is substantial cross-country variation in terms of inflation rates and monetary accommodation by central banks, this methodology is useful for identifying the impact of monetary policy on inflation volatility. One should, however, be cautioned to extend this directly to other contexts. For instance, Okun's methodology is not as useful for analyzing the dynamic impact of monetary policy changes on inflation volatility in an economy.

In the 1980s the emphasis shifted towards measuring inflation uncertainty. This shift was partly due to the understanding that the costs of unanticipated inflation are more significant than those of anticipated inflation, and partly due to the recognition that earlier volatility measures did not allow for the identification of the unanticipated component of inflation volatility. See, for example, Demetriades (1989), Cukierman and Wachtel (1979, 1982), Fischer (1981), Lahiri and Teigland (1987), Pagan *et al.* (1983), and Zarnowitz and

[2]See also Foster (1978), and Logue and Willet (1976) who reached to similar conclusions using annual data.

Lambros (1987). In these studies, the measures of inflation uncertainty were based either on econometric forecasting models, or on responses of forecasters to survey questionnaires. When forecast errors of structural or ARIMA models of inflation are used as a measure of inflation uncertainty, the results appear to be sensitive to model specification. Survey-based inflation forecasts enable researchers to circumvent such problems.

However, estimates of inflation uncertainty from survey-based studies also differ significantly, since some studies use data from direct surveys, while others use data from indirect surveys. In direct surveys, from each forecaster surveyors collect the subjective probability distribution of his/her forecast, and hence the variance of the forecast of each forecaster can be readily computed.[3] In indirect surveys, from each forecaster surveyors collect his/her point forecast, and thus the variance of forecasts of all the forecasters can be computed.[4] In direct and indirect survey studies, the dispersion of these forecasts is used as a measure of inflation uncertainty. Since these measures from survey data capture different aspects of inflation uncertainty (such as disagreement), they should be treated differently.

Some authors have also specified conditional variances of forecasting errors obtained from ARCH/GARCH models as the one-step-ahead uncertainty. See, for example, Engle (1983), Evans (1991), and Brunner and Hess (1993). These models are used to capture nonlinearity in the formation of expectations. There is, however, no agreement on which model should be used to forecast inflation [see, e.g., Cecchetti (1995)]. Differences in model specification ultimately lead to different conclusions about the effect of inflation level on inflation uncertainty [see, e.g., Crawford and Kasumovich (1996)].

We believe that there is an additional limitation associated with this methodology for the purposes of policy evaluation. Once the one-step-ahead uncertainty measures are obtained from an ARCH/GARCH model, the next step typically is to calculate the contemporaneous correlation between the inflation rates and their conditional variances. As mentioned in the introduction, however, the correlation of this kind cannot account for

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[3]Examples of such surveys in the U.S. are the quarterly surveys of economic forecasters of the American Statistical Association, and the National Bureau of Economic Research.

[4]Examples of such surveys in the U.S. are the Michigan Social Research Council Surveys and the Livingston Survey of Economic Forecasters. In Canada, the Conference Board of Canada conducts similar annual surveys.

the impact of a monetary policy change that usually takes months to have a full impact. This limitation becomes particularly prevalent when higher frequency data are used.

It should also be mentioned that, uncertainty measures based on econometric modelling and survey data typically yield very different results for the same sample period. Recently, for example, Batchelor and Dua (1996) studied a wide range of volatility and uncertainty measures, and found that they are not highly correlated over the same sample period. Therefore, in light of the more recent research, the observation made by Khan (quoted above) remains, by and large, valid even today. We now turn to a discussion of volatility measures that are suitable for our purposes and are used in our empirical analysis.

2.2 Local Volatility

In this paper we adopt two local volatility measures; namely, the centered moving averages of variances, and mean absolute deviations of actual inflation. These measures are closely related to the ones used by Khan (1977) and Klein (1977). These authors initially adopted this method to deal with possible nonstationarity in the inflation data, and used a k-term moving standard deviation of inflation, which can be considered as a local volatility measure. The most important feature of this method is that it invokes the assumption of constant mean over k periods. We believe that these measures are suitable for exploring the relationship between the inflation rate and its volatility with particular emphasis on monetary policy changes. Since the impact of monetary policy changes on inflation rate is materialized gradually over time, and inflation series exhibit stationarity in the short-run, this approach allows us to study the effects of monetary policy on inflation variability. It is also a reasonable starting point for the evaluation of the monetary policy in Canada that aims at low and stable inflation rates.

Although it may be argued that the local volatility measures may not capture explicitly how information is updated, they do have the advantage of avoiding a model-specific information-updating mechanism. Although local volatility measures may not distinguish anticipated from unanticipated inflation variability, they can be interpreted as a way to capture unanticipated inflation variability when local volatility and unanticipated volatility are positively correlated.[5] This interpretation is consistent with the view that high volatility of inflation may increase the noise to signal ratio in an uncertain environment, and may therefore impede inflation forecasts [see, e.g., Balvers and Cosimano (1994)].

In particular, we consider the local variance of inflation rates (ω_t^2), and the local mean absolute deviation of inflation rates ($|\omega_t|$). We calculate ω_t^2 and $|\omega_t|$ using a window approach, which allows to compute the volatility in the vicinity of each observation on inflation. For each window width specified by an integer J, the local variance of inflation rates at time t, $\omega_t^2(J)$, is defined as

$$\omega_t^2(J) = \frac{1}{J}\sum_{j=0}^{J}\left[\pi_{t+j-J/2} - \bar{\pi}_t\right]^2, \tag{1}$$

where $\bar{\pi}_t = (\sum_{j=0}^{J}\pi_{t+j-J/2})/J$, and π_t is the inflation rate at time t. Similarly, the local mean absolute deviation of inflation rates at time t, $|\omega_t|(J)$, is defined as

$$|\omega_t|(J) = \frac{1}{J}\sum_{j=0}^{J}|\pi_{t+j-J/2} - \bar{\pi}_t|. \tag{2}$$

Using equations (1) and (2) we calculate local volatility measures with different window widths. To investigate the relationship between the inflation rate and the local volatility ω_t^2 (similarly for $|\omega_t|$), we consider a quadratic econometric model of the form

$$\omega_t^2 = \alpha_0 + \alpha_1\pi_t + \alpha_2\pi_t^2 + \varepsilon_t, \tag{3}$$

where the window widths are suppressed, and ε_t are random disturbances. This model captures possible nonlinearity that may exist between inflation and its volatility.

To demonstrate the robustness of our results, we also adopt a nonparametric regression to estimate the relationship between the inflation rate and its local volatility. This method has the advantage of imposing minimum structure on the data, and is particularly useful for estimating highly nonlinear relationships (Härdle 1990). More specifically, the

[5] For the purposes of comparison, following Evans (1991), we also examine inflation uncertainty using a Kalman filter-ARCH model in section 3.

nonparametric regression for ω_t^2 (similarly for $|\omega_t|$),

$$\omega_t^2 = m(\pi_t) + \varepsilon_t, \tag{4}$$

allows for a flexible functional form $m(\pi_t)$ linking inflation rate to its volatility. Our kernel estimate of the function $m(\pi_t)$ is given by:

$$\hat{m}_T(x) = \frac{1}{Th} \sum_{t=1}^{T} \left(\frac{K_h(x - \pi_t)}{\hat{f}_T(x)} \right) \omega_t^2, \tag{5}$$

where T is the number of observations, h is the bandwidth parameter, K_h is the Gaussian kernel function with bandwidth h, and $\hat{f}_T$ is the corresponding empirical density function computed by:

$$\hat{f}_T(x) = \frac{1}{Th} \sum_{t=1}^{T} K_h(x - \pi_t). \tag{6}$$

In the calculations the bandwidth parameter is specified as $h = \hat{\sigma}_\pi \, [4/(3T)]^{.2}$, where $\hat{\sigma}_\pi^2$ is the sample variance of the data [see Silverman (1986), p. 45].

3 Data and Results

3.1 A First Look at the Data

The monthly data used in this study for the period from 1975:1 to 1995:12 are obtained from Statistics Canada's CANSIM database. Our choice of the sample period is primarily determined by the available consistent series on a number of variables used in the analysis. Since we want to minimize the impact of supply effects on the variability of inflation, in the analysis we focus on the seasonally unadjusted consumer price sub-index that excludes food and energy [denoted by CPI (ex. food)].[6] Monthly inflation rates are calculated by first log-differencing the price index series.

[6] Monthly data with 1986 base year (CANSIM matrix number P700285) are used. In our empirical analysis, we have also experimented with (i) the seasonally adjusted CPI (ex. food), and (ii) the overall CPI which includes food and energy. We found that our results are not affected.

Table 1 reports the summary statistics for the inflation rate, the growth rate of real output, and the growth rates of nominal and real M2.[7] To capture the effects of different monetary policy regimes on inflation and its volatility, we divide the sample into two subperiods: from 1975:1 to 1987:12, and from 1988:1 to 1995:12. We take 1988 as the beginning of the Bank of Canada's price stability policy. It is interesting to note that although average inflation rate in Canada declined during the period from 1988 to 1995, as shown in table 1, its variability measured by the standard deviation of the inflation rate *increased.*

Of course, the variability of inflation is affected by a host of variables, including monetary policy. For instance, Table 1 provides summary statistics on real output and nominal money growth rates and shows that both were lower and less volatile in the second subperiod. Changes in the variability of output or money growth do not coincide with the observed higher variability of inflation from 1988 to 1995. The inflation rate is variable when real and monetary variables grow at a *higher* rate. Variability in these variables is not associated with increased variability of inflation.

The frequency and persistence of the monetary regime shifts may also affect the observed volatility of inflation. To capture these effects on inflation volatility, in this study, we use the spread between the overnight lending rate and the short-term interest rate as the indicator of monetary policy changes. According to Laidler and Robson (1993, pp.77-78) this interest rate spread is a "useful indicator of the Bank of Canada's intentions." For instance, a rising interest rate spread results from the rising overnight lending rate with respect to the short-term interest rate, and this is indicative of tight monetary policy.[8] Figure 1 shows the behaviour of interest rate spread over the sample period.

We note that the variability of inflation after 1988 may be significantly influenced by the introduction of the federal goods and services tax (GST) on January 1, 1991. Given the importance of coordination between the monetary and fiscal policies, it is a common practice not to discount the impact of indirect taxes on CPI inflation. Therefore, we focus on the results pertaining to the CPI inflation that includes indirect taxes, and consider

[7] M2 is used as the measure of money supply because the definition of M1 has been modified over the years in Canada.

[8] See also Bank of Canada (1991), and Clinton (1991) for similar interpretations.

the GST effects in our empirical analysis.

3.2 Window Estimates of Volatility

In this paper we ask whether there exits a monotone positive relationship between the level of inflation and its volatility. To this end, first we estimate the measures of local volatility using equations (1) and (2) for four different window widths, $J = 3, 6, 9$, and 12. Then a regression analysis is used to examine if the inflation level and its local volatility have a nonlinear relationship. For brevity, we discuss the results of local volatility measured by the variance of inflation, and present the results of local volatility measured by the mean absolute deviation in Appendix A.

The regression results of equation (3) are presented in Table 2. The estimates suggest that the relationship between inflation and its volatility is indeed nonlinear, and that the volatility of inflation reaches its minimum at low but positive levels of inflation. In particular, in all cases the coefficient estimates of α_2 are positive and significant. These are indicative of the U-shape relationship between the level of inflation and its volatility. In other words, inflation volatility appears to increase at low and negative levels of inflation, as well as at high levels of inflation. Based on these estimates we calculate the level of inflation that minimizes the volatility of inflation and find that local volatility of inflation, as measured by the variance $\omega_t^2(3)$, and mean absolute deviation $|\omega_t(3)|$, is minimized at around 4.5%, and 2.2% annual inflation rate, respectively.

We note that there is a large increase in volatility around the time when the GST was introduced in January 1991; see Figure 2. To control the impact of the GST, we add a dummy variable into equation (3). Table 2 shows that the coefficient estimate of the dummy variable is positive and significant at the one percent level. However, we still cannot reject the U-shape relationship between the level of inflation and its volatility centered around moderate rates of inflation.

To confirm the robustness of these results we also adopt the nonparametric regression model specified in equation (4). In order to facilitate the presentation, we display the results from the nonparametric regression in graphs. Figure 3 shows the link between actual inflation and local volatility, and the link between actual inflation and the fitted values of local volatility obtained from the nonparametric regression for different window widths and for the two subperiods. The extreme observation on the upper right corner of each picture pertains to 1991:1, which corresponds to the introduction of the GST discussed earlier. Again the U-shape pattern emerges from these estimates, and the results show that high local volatility is associated with both low and high inflation rates. This pattern is robust to different measures of local volatility [i.e., variance, equation (1), and absolute deviation, equation (2)[9]], as well as various window widths. Overall, zero inflation is not the level of inflation at which its volatility is minimized. Instead, the volatility of inflation reaches to its lowest point at a moderate inflation rate. The nonparametric regression results suggest that both local volatility measures of inflation, variance $\omega_t^2(3)$, and mean absolute deviation $|\omega_t(3)|$, are minimized at 2.4% annual inflation rate, and these are in conformity with those that emerge from the parametric regression.

In order to account for the GST effect on inflation volatility, we again add a dummy variable into equation (4). This results in an almost "flat" relationship between inflation and its local volatility, again going against the belief of a positive monotonic relationship; see Figure 4.

To examine how volatility measures are related to the indicators of monetary policy change, we calculate the correlation between the interest rate spread (an indicator of the monetary policy) and the local volatility measures of inflation. Table 3 shows the sample correlations between local volatility (as measured by the variance) with different window widths and the interest rate spread. The sample correlations between monetary policy changes and inflation volatility are low and not significant for the period from 1975 to 1987. On the other hand, the correlations, in particular for the window width 3, are strong and significant during the period from 1988 to 1995. These results suggest that contractionary monetary policies (a positive interest rate spread) were positively correlated with volatile inflation rates during the inflation targeting period of 1988-1995. Clearly, the high volatility of inflation during the recent disinflation period in Canada does exist and this has significant policy implications.

[9]These graphs are provided in Appendix A.

3.3 The Kalman Filter–ARCH Model

Since uncertainty measures derived from model-based expectations have been widely used in the recent literature, it may be useful to provide some comparison of our results with these studies. We follow Evans (1991), and combine the Kalman filter and ARCH specification to accommodate a wide range of expectation formation mechanisms. The advantage of this framework is that it allows for both time-varying state parameters and time-varying conditional variances. The model specifies a time-varying Kalman filter for the mean (inflation) equation, and an ARCH model for the variance (uncertainty) equation.

Since this methodology is becoming standard, here we only present the specification of the inflation process and present the details in Appendix B. Following Evans (1991), we specify the inflation rate (π_t) as an autoregressive process

$$\pi_{t+1} = \mathbf{x}_t \beta_{t+1} + \xi_{t+1}, \tag{7}$$

where $\mathbf{x}_t = [1, \pi_t, \ldots, \pi_{t-k}]$, and ξ_{t+1} are random disturbances. Note that in this specification the parameters β_t are allowed to vary over time so that

$$\beta_{t+1} = \beta_t + V_t,$$

where V_t are random disturbances.

We employ the likelihood ratio test to select the appropriate specification of $\mathbf{x}_t$ in the mean equation (7). Two specifications, $\mathbf{x}_t = [1, \pi_t, \pi_{t-1}]$ and $\mathbf{x}_t = [1, \pi_t]$, are compared. Since we cannot reject the restricted specification of equation (7) with $\mathbf{x}_t = [1, \pi_t]$, we adopt this parsimonious model. The estimation results are reported in Table 4. The state parameter estimates based on all of the sample information are significant at the one percent level.

As discussed in Appendix B, we compute "prediction errors" or one-step-ahead uncertainty as a way to gauge uncertainty. The prediction errors are computed by using only the past and current information. We then analyze the unexplained changes in the inflation rate using ARCH models.

Estimation results of the ARCH model based on prediction errors obtained from the

Kalman filter are reported in Table 5. The results indicate that there is no ARCH(1) or ARCH(2) effect in prediction errors.[10] However, the association between inflation and inflation uncertainty in Canada during this period can still be examined by adopting the prediction errors as a measure of one-step-ahead uncertainty. We analyze this relationship by exploiting the correlations between this uncertainty measure and four leads and lags of inflation. Table 6 shows that the correlation is significant and positive only between the *contemporaneous* prediction error and the inflation rate. But, as argued above, due to the time lag between a monetary policy shift and materialization of its effects on inflation, this correlation can not be taken as evidence for positive association between inflation and its volatility induced by the policy changes. In addition, the correlation coefficient estimates between prediction errors and lagged inflation are *negative*. Also as the number of lags increases, the absolute value of the correlation coefficient estimate between prediction errors and lagged inflation decreases. The test for independence indicates that the inflation rates and the prediction errors are statistically independent at all lags and leads.[11] These empirical results support our earlier arguments against using one-step-ahead forecast errors to measure inflation volatility for policy analysis.

To explore the link between uncertainty based on the Kalman filter estimates and monetary policy changes, we also estimate the correlations between prediction errors and current and lagged interest rate spreads, and test their statistical significance. Table 7 reports the results of this analysis. None of the estimated correlation coefficients are significant. Therefore, our results suggest that uncertainty measures constructed from such regression models are unlikely to capture the relationship between the model-based measure of inflation volatility and policy indicators. As such, they may be uninformative for monetary policy analysis. We note that, unlike the model-based uncertainty measures, the local volatility measures we advocate are significantly correlated with the interest rate spreads, compare Tables 3 and 7.

[10]These are consistent with earlier studies on the U.S. inflation which find weak or no ARCH effect; see, e.g., Cosimano and Jansen (1988), and Evans (1991).

[11]See Lehmann (1994, p.251) for a detailed discussion of this test. The test for independence has a high power under relatively weak conditions.

4 Theoretical Considerations

How can we account for the observed high inflation volatility during the recent disinflation period in Canada? In this section, we discuss whether some rational expectations model of disinflation can provide insights. In particular, we consider the staggered contract models of Phelps (1978) and Taylor (1980), as well as policy credibility arguments to explain the existence of higher volatility at lower inflation rates.

In the macroeconomics literature, one of the most frequently used sticky price models based on rational expectations is the staggered contract model of Phelps and Taylor. To evaluate the implications of this model with regard to the relationship between the level of inflation and its volatility, we extend Talyor's (1980, 1981) analysis by specializing in a two-period-contracts version of the model. In particular, we specify a monetary policy rule which targets inflation and discuss the implications of a "drastic" policy, which we define below.

In what follows, all variables are stated in logs. Assume that the growth rate of money supply at time t, Δm_t, is given by

$$\Delta m_t = \theta \left[E(\pi_{t+1}|I_t) - \pi^*_{t+1} \right] + u_t, \tag{8}$$

where Δ is the first difference operator; $E(\pi_{t+1}|I_t)$ is the expected inflation at time $t+1$ given the information set I_t at time t; π^*_{t+1} is the inflation target at time $t+1$, set by the monetary authority at time t; and u_t is the white noise with variance σ^2_u. We will model the target setting process below. Although in practice money supply rules are rarely used as monetary instruments, equation (8) may be interpreted as an intermediate target. Essentially equation (8) relates the discrepancy between the expected future inflation and inflation target to the monetary policy rule. Note that the parameter θ in equation (8) can take a range of values with different interpretations. The sign of θ indicates the direction that the policy is taking: a positive θ indicates that the policy reinforces the rational expectations vis-à-vis the preset target. For example, if the target exceeds the rational expectations about inflation, $E(\pi_{t+1}|I_t) < \pi^*_{t+1}$, the money supply m_t tends to decrease. This will result in a decreased actual inflation at time $t+1$. Conversely, a negative θ indicates that the policy reinforces the target vis-à-vis the rational expectations about inflation. If, for example, expectations exceed the target, $E(\pi_{t+1}|I_t) > \pi^*_{t+1}$, the money supply m_t tends to decrease. This will result in a decreased actual inflation at time $t+1$.

Following the model of Taylor (1980), we specify inflation as an autogressive process:

$$\pi_t = \alpha\pi_{t-1} + \frac{1}{2}(\varepsilon_t + \varepsilon_{t-1}), \tag{9}$$

where $0 < \alpha < 1$; the weighted sum of the residuals $\frac{1}{2}(\varepsilon_t + \varepsilon_{t-1})$ reflects the two-period nature of the staggered contracts; and ε_t is the white noise with variance σ^2_ε.

The third equation in the model is the quantity of money equation:

$$\Delta y_t = \Delta m_t - \pi_t + v_t, \tag{10}$$

where y_t is real output, and v_t is the white noise with variance σ^2_v.

Taking the expectation of equation (9) and invoking rational expectations give

$$E(\pi_{t+1}|I_t) = \alpha\pi_t. \tag{11}$$

Since the actual inflation and money supply rule are closely linked, the inflation targets are important in this analysis. In what follows, we will characterize different target setting schemes using a simple framework. We assume that the inflation target at time $t+1$ is a multiple (λ) of expected inflation at time $t+1$:

$$\begin{aligned} \pi^*_{t+1} &= \lambda E(\pi_{t+1}|I_t) \\ &= \lambda\alpha\pi_t. \end{aligned} \tag{12}$$

In equation (12), as λ differs from one in absolute value, the inflation targeting policy becomes more "drastic," because the target departs from the rationally expected inflation rate. To see the implications of this targeting scheme and drastic policy measures on inflation variability, substitute equations (8), (11) and (12) into equation (10), and rearrange to obtain:

$$\pi_t = \frac{1}{[\theta\alpha(1-\lambda) - 1]}[\Delta y_t + v_t + u_t]. \tag{13}$$

Thus, the variance of inflation σ_π^2 is calculated as:

$$\sigma_\pi^2 = \frac{1}{[\theta\alpha(1-\lambda)-1]^2}[\sigma_{\Delta y}^2 + 2\text{cov}(\Delta y, v+u) + \sigma_v^2 + \sigma_u^2]. \tag{14}$$

It can be readily verified that, depending on the sign and magnitude of θ, a drastic policy (i.e., a "large" $|\lambda - 1|$) can induce higher inflation volatility. Take, for example, the case in which the money supply rule reinforces the rational expectations with respect to the set targets with $\theta > 0$ in equation (8). When inflation expectations are below the target, and the target is set as a smaller fraction of these expectations with $\lambda < 1$ in equation (12), then lower the value of λ higher is the variance of inflation in equation (14). This is because the multiplier $(1/[\theta\alpha(1-\lambda)-1]^2)$ in equation (13) increases as λ decreases; see Table 8 for a demonstration. The increased variance in inflation in this case is primarily due to "over-reaction" by the monetary authorities to inflation in setting the target. We conjecture that this case may help explain the increased inflation volatility in the recent Canadian disinflation experience. In their influential study on Canadian disinflation, Laidler and Robson (1993) make a similar point.[12]

Why might policymakers over-react to inflation? We consider two issues. First, when policymakers step into new territories they may over-react to current inflation because of uncertainties. Second, credibility concerns of the policymakers may also lead to over-reaction. With regard to the first issue, Balvers and Cosimano (1994) recently considered a model in which the inflation volatility may be higher when the monetary policy moves into an "unknown territory." Hence, the policymakers may implement a drastic monetary policy in an attempt to reduce the level of inflation and this may lead to higher inflation volatility.

With regard to the second issue, a large and growing literature in macroeconomics and political economy suggests that credibility of the policymaker is an important determinant of policy effectiveness. When a policy is not credible, the public tends to pay less attention to policymakers' intentions, and more attention to past performance of the

[12] Another interesting example occurs when the money supply rule reinforces the inflation target with respect to expected inflation with $\theta < 0$ in equation (8). When inflation expectations exceed the target, and the target is set as a larger multiple of the inflation expectations with $\lambda > 1$ in equation (12), then higher the value of λ lower the variance of inflation in equation (14); see also Table 8 for a demonstration.

policymakers. For instance, Cukierman and Meltzer (1986, Proposition 5) argue that the volatility of inflation rate is higher in those countries and periods in which policymakers lack credibility. Therefore, in order to reinstate their credibility policymakers may have the incentive to over-react to inflation. In the game-theoretic literature this corresponds the case where policymakers act tougher than necessary to convince the public about their intentions of reducing inflation; see, e.g., Backus and Driffill (1985).

5 Conclusion

A common belief among economists is that the level of inflation and the variability of inflation are positively correlated. This paper has examined the relationship between the level of inflation and its volatility in Canada from 1975 to 1995. However, we cannot find any supporting evidence for the presumed positive correlation. Despite the significant decline in the level of inflation after the Bank of Canada's implementation of a price stability policy in 1988, there is little evidence indicating a significant decline in the volatility of inflation. This finding appears to be robust to alternative measures of local volatility and uncertainty. Of course, we do not contend that high volatility of inflation is a property of low inflation, but underscore the possibility that such an outcome may arise from a change in monetary policy regime. In particular, during a transition from high to low inflation the policymaker may have to cope with the uncertainties associated with the new environment, and may have incentives to over-react to inflation to maintain its credibility. But, the finding that high volatility of inflation can coexist with low levels of inflation poses a challenge to the conventional belief that zero inflation minimizes the costs of inflation volatility. We believe that there are a number of benefits to stabilizing inflation at a low rate, and low inflation volatility appears to be one of them. However, there is very little empirical evidence to suggest that zero inflation is optimal in reducing inflation volatility.

It may be argued that higher volatility in the recent Canadian disinflation is a transitory effect, and that once the disinflation policies are fully incorporated into expectations the volatility will dampen. This is a possibility that we do not rule out. However, our findings suggest that volatility would not necessarily be lower at zero inflation.

References

Backus, D. and T. Driffill. (1985). "Rational expectations and policy credibility following a change in regime," *Review of Economic Studies* 52: 211-221.

Balvers, R.J., and T.F. Cosimano. (1994). "Inflation variability and gradualist monetary policy," *Review of Economic Studies* 61: 721-738.

Bank of Canada. (1991). "The implementation of monetary policy in a system with zero reserve requirements," Bank of Canada Discussion Paper No. 3.

Batchelor, R., and P. Dua. (1996). "Empirical measures of inflation uncertainty: A cautionary note," *Applied Economics* 28: 333-341.

Brunner, A., and G. Hess. (1993). "Are higher levels of inflation less predictable? A state-dependent conditional heteroscedasticity approach," *Journal of Business and Economic Statistics* April: 187-197.

Cecchetti, S.G. (1995). "Inflation indicators and inflation policy," *NBER Macroeconomics Annual 1995*: 189-219.

Clinton, K. (1991). "Bank of Canada cash management: the main technique for implementing monetary policy," *Bank of Canada Review* January.

Cosimano, T.F., and D. Jansen. (1988). "Estimates of the variance of U.S. inflation based on ARCH model: A comment," *Journal of Money, Credit, and Banking* 20: 409-421.

Crawford, A. and M. Kasumovich. (1996). "Does inflation uncertainty vary with the level of inflation?" Bank of Canada Working Paper, 96-9.

Crow, J.W. (1988). "The work of Canadian monetary policy," *Bank of Canada Review*, February.

Cukierman, A., and P. Wachtel. (1979). "Differential inflationary expectations and the variability of the rate of inflation: Theory and evidence," *American Economic Review* 69: 595-609.

Cukierman, A., and P. Wachtel. (1982). "Inflationary expectations: Reply and further thoughts on inflation uncertainty," *American Economic Review* 72: 508-512.

Cukierman, A., and A.H. Meltzer. (1986). "A theory of ambiguity, credibility, and inflation uncertainty under discretion and asymmetric information," *Econometrica* 54: 1099-1128.

Demetriades, P.O. (1989). "The relationship between the level and variability of inflation: Theory and evidence," *Journal of Applied Econometrics* 4: 239-250.

Engle, R.F. (1982). "Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflation," *Econometrica* 50: 987-1007.

Engle, R.F. (1983). "Estimates of the variance of the U.S. inflation based on the ARCH model," *Journal of Money, Credit, and Banking* 15: 286-301.

Evans, M. (1991). "Discovering the link between inflation rates and inflation uncertainty," *Journal of Money, Credit, and Banking* 23: 169-184.

Fischer, S. (1981). "Towards an understanding of the costs of inflation II," *Carnegie-Rochester Conference Series on Public Policy* 15: 5-41.

Fortin, P. (1996). "Presidential address: The great Canadian slump," *Canadian Journal of Economics* 29: 781-787.

Foster, E. (1978). "The variability of inflation," *Review of Economics and Statistics* 60: 346-350.

Friedman, M. (1977). "Nobel lecture: Inflation and unemployment," *Journal of Political Economy* 85: 451-472.

Härdle, W. (1990). *Applied Nonparametric Regression.* New York: Cambridge University Press.

Harvey, A.C. (1981). *Time Series Models.* Oxford: Philip Allan Publishers.

Howitt, P. (1990). "Zero inflation as a long-term target for monetary policy," in *The Goal of Price Stability*, ed. by Richard G. Lipsey, pp. 67-108, Toronto: C.D. Howe Institute.

Khan, M.S. (1977). "The variability of expectations in hyperinflations," *Journal of Political Economy* 85: 817-827.

Klein, B. (1977). "The demand for quality-adjusted cash balances: Price uncertainty in the U.S. demand for money function," *Journal of Political Economy* 85: 691-715.

Lahiri, K., and C. Teigland. (1987). "On the normality of probability distributions of inflation and GNP forecasts," *International Journal of Forecasting* 3: 269-279.

Laidler, D.E.W., and W.B.P. Robson. (1993). *The Great Canadian Disinflation: The Economics and Politics of Monetary Policy in Canada.* Toronto: C.D. Howe Institute.

Lehmann, E. L. (1994). *Testing Statistical Hypotheses.* New York: Chapman & Hall.

Logue, D.E., and T.D. Willet (1976). "A note on the relation between the rate and variability of inflation," *Economica* 43: 151-158.

Okun, A. (1971). "The mirage of stable inflation," *Brookings Papers on Economic Activity* 2: 485-498.

Pagan, A.R., A.D. Hall, and P.K. Trivedi. (1983). "Assessing the variability of inflation," *Review of Economic Studies* 50: 585-596.

Phelps, E.S. (1978). "Disinflation without recession: Adaptive guideposts and monetary policy," *Weltwirtschaftliches Archiv* 100: 239-265.

Silverman, B.W. (1986). *Density Estimation.* New York: Chapman & Hall.

Taylor, J.B. (1980). "Aggregate dynamics and staggered contracts," *Journal of Political Economy* 88: 1-24.

Taylor, J.B. (1981). "On the relation between the variability of inflation and the average inflation rate," *Carnegie-Rochester Conference Series on Public Policy* 15: 57-85.

Zarnowitz, V., and L.A. Lambros (1987). "Consensus and uncertainty in economic prediction," *Journal of Political Economy* 95: 591-621.

Table 1: Summary Statistics

Monthly data: 1975:1-1995:12

	1975:1-1987:12		1988:1-1995:12	
Variable	Mean	Std. Deviation	Mean	Std. Deviation
Inflation	.563	.324	.265	.344
Real output growth	.302	.740	.132	.432
Nominal M2 growth	.934	.564	.529	.451
Real M2 growth	.341	.606	.284	.487

Note: Means and standard deviations are multiplied by 100. Output is total business sector gross domestic product. Inflation is the monthly CPI inflation, excluding food and energy. Real M2 is M2 deflated by CPI.

Table 2: Estimates of the Relationship between the Inflation Rate and Its Volatility Measured by Variance

Dep. Var.: Variance of Inf. — Monthly Data: 1975:1-1995:12

Window Width J	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\hat{\alpha}_2$	$\hat{\delta}$	R^2
3	**.0001 (.0000)	**-.0008 (.0003)	**.1156 (.0214)	-	.1259
6	**.0001 (.0000)	*-.0005 (.0002)	**.0567 (.0158)	-	.0562
9	**.0001 (.0000)	-.0003 (.0002)	**.0339 (.0130)	-	.0291
12	**.0000 (.0000)	-.0003 (.0001)	*.0265 (.0109)	-	.0261
3	**.07 (.005)	*-.0003 (.0001)	**.0488 (.0117)	**.4 (.020)	.7554
6	**.07 (.004)	-.0002 (.0001)	**.0277 (.0086)	**.2 (.010)	.7248
9	**.07 (.003)	-.0001 (.0001)	**.0168 (.0076)	**.2 (.007)	.6709
12	**.07 (.003)	-.0000 (.0001)	*.0141 (.0067)	**.1 (.006)	.6268

Note: Standard errors are in parentheses. The beginning and end of the samples vary with the window width. This table reports the OLS estimates of the regression equations

$$\omega_t^2(J) = \alpha_0 + \alpha_1 \pi_t + \alpha_2 \pi_t^2 + \epsilon_t,$$

$$\omega_t^2(J) = \alpha_0 + \alpha_1 \pi_t + \alpha_2 \pi_t^2 + \delta \, Dummy_{Jt} + \epsilon_t,$$

where $\omega_t^2(J)$ is defined in equation (1), $Dummy_{Jt}$ is the dummy variable that takes value one for $t=$ 1991:1-J, ..., 1991:1+J, and zero otherwise, and π_t is the inflation rate. R^2 is the coefficient of determination of the regression. When the dummy variable is included, coefficient estimates and standard errors of α_0 and δ are multiplied by 10^4, to reduce the number of leading zeros.

** Coefficient estimate is different from zero at the one percent level.

* Coefficient estimate is different from zero at the five percent level.

Table 3: Correlations between Interest Rate Spread and Local Volatility Measured by Variance

Monthly data: 1975:1-1995:12

	1975-1995		1975-1987		1988-1995	
Window Width, J	Corr.	p-value	Corr.	p-value	Corr.	p-value
3	.0385	.2733	-.0102	.5495	.1880	.0372
6	.0729	.1299	.0612	.2322	.2490	.0112
9	.0312	.3169	-.0241	.6110	.2487	.0140
12	.0461	.2438	.0254	.3855	.2075	.0392
Average inflation	.4490		.5632		.2645	

Note: Inflation is the log difference of monthly CPI inflation, excluding food and energy, multiplied by 100. Volatility is $\omega_t^2(J)$ as defined in equation (1). Since the volatility measure includes lead and lag values of inflation sample sizes vary across window widths. Sample periods are non-overlapping. p-values are for the test for independence. The test statistic is $\sqrt{T_J}\rho(J)$, and has an asymptotic standard normal distribution. T_J is the number of observations for window width J, and $\rho(J)$ is the estimate of the correlation coefficient.

Table 4: Estimates of the Kalman Filter Model

Monthly Data: 1975:3-1995:12

Measurement Equation:	$\pi_{t+1} = \beta_{0,t} + \beta_{1,t}\pi_t + \xi_t, \quad t = 1, \ldots, T.$
Transition Equations:	$\beta_{0,t+1} = \beta_{0,t} + V_{0,t}, \quad t = 1, \ldots, T.$
	$\beta_{1,t+1} = \beta_{1,t} + V_{1,t}, \quad t = 1, \ldots, T.$

Variable	$\beta_{0,T}$	$\beta_{1,T}$
Coefficient Estimate	-.0006	-.3477
Standard Error	.0016	.0626
t-statistic	-.4005	-5.5520

Note: For initialization, default identity matrices are used. The coefficients are estimated using $T = 250$ observations. Other state parameter estimates for $t < T$ are not reported. The likelihood ratio tests were used to select the AR(1) specification as opposed to AR(2) and AR(3).

Table 5: Lagrange Multiplier Tests for ARCH(1) and ARCH(2) on Residuals from the Kalman Filter

	p-value	
Variable	ARCH(1)	ARCH(2)
Prediction Error	.25568	.50743

Note: The reported test statistic is TR^2, where T is the number of observation, and R^2 is the coefficient of determination of the regression of the squared error term on a constant and the lagged squared error term(s). TR^2 follows a χ^2 distribution with 1 or 2 degree(s) of freedom.

Table 6: Correlations between the Prediction Errors of the Kalman Filter and Inflation Rates

Lead (+) and Lag (-) Inflation Rate (k)	Correlation Coefficient ($\rho(k)$)	p-value
-4	-.008029	.55021
-3	-.065793	.84944
-2	-.315310	1.00000
-1	-.179420	.99760
0	*.698350	.00000
1	-.038950	.72855
2	-.025562	.65516
3	.032904	.30363
4	.051895	.20879

Note: Prediction errors are obtained from an AR(1) inflation equation with time-varying parameters using all the past and current sample information. p-values are for the test for independence. The test statistic is $\sqrt{T}\rho(k)$, and has an asymptotic standard normal distribution. T is the number of observations, and $\rho(k)$ is the estimate of the correlation coefficient with k lags(s) or lead(s).

* Coefficient estimate is different from zero at the five percent level.

Table 7: Correlations between the Prediction Errors and the Interest Rate Spread

Monthly data: 1975:1-1995:12

No. of Lags on the interest rate spread (k)	1975-1995 Corr.	1975-1995 p-value	1975-1987 Corr.	1975-1987 p-value	1988-1995 Corr.	1988-1995 p-value
4	-.0046	.5288	.0018	.4911	-.0341	.6310
3	-.1231	.9737	-.1315	.9476	-.1569	.9379
2	-.0846	.9086	-.0795	.8364	-.1524	.9324
1	.0470	.2295	.0589	.2340	.0236	.4085
0	.0048	.4701	-.0211	.6028	.1118	.1367

Note: Prediction errors are obtained from an AR(1) inflation equation with time-varying parameters using all the past and current sample information. p-values are for the test for independence. The test statistic is $\sqrt{T}\rho(k)$, and has an asymptotic standard normal distribution. T is the number of observations, and $\rho(k)$ is the estimate of the correlation coefficient with k lags(s) or lead(s).

Table 8: The Relationship between the Variance of Inflation and Targeting Rule

	$\frac{1}{[\theta\alpha(1-\lambda)-1]^2}$ $\alpha = .8$	
λ	$\theta = .5$	$\theta = -.5$
0.2	2.16	0.69
0.4	1.73	0.76
0.6	1.42	0.83
0.8	1.18	0.91
1.0	1.00	1.00
1.2	0.86	1.11
1.4	0.74	1.23
1.6	0.65	1.38
1.8	0.57	1.56
2.0	0.51	1.78

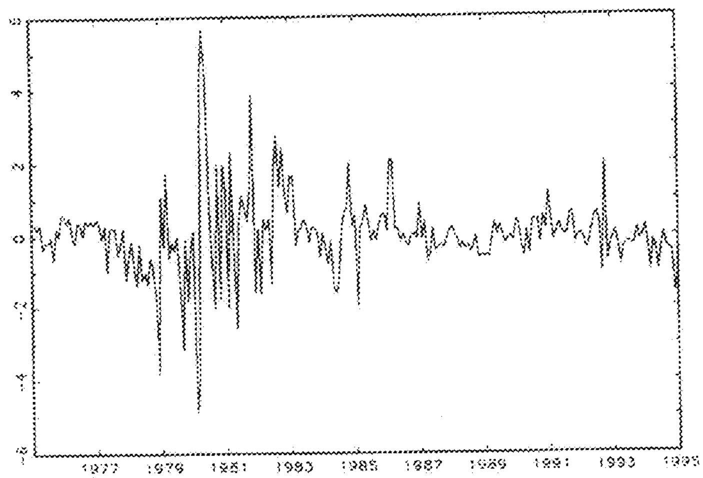
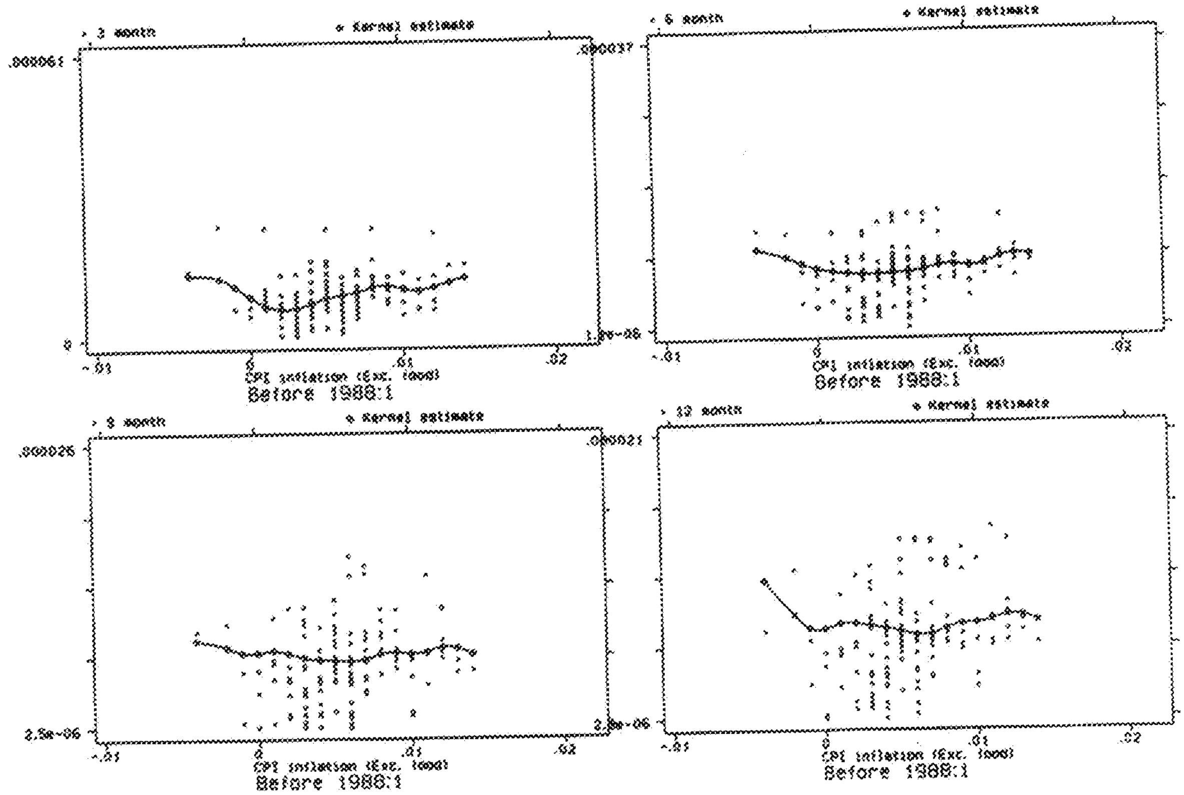


Figure 1: Spread between the Overnight Lending Rate and Short-Term Interest Rate



a) Variance (1975:1-1987:12)

Figure 2: Inflation and Inflation Volatility in Canada, 1975-1995

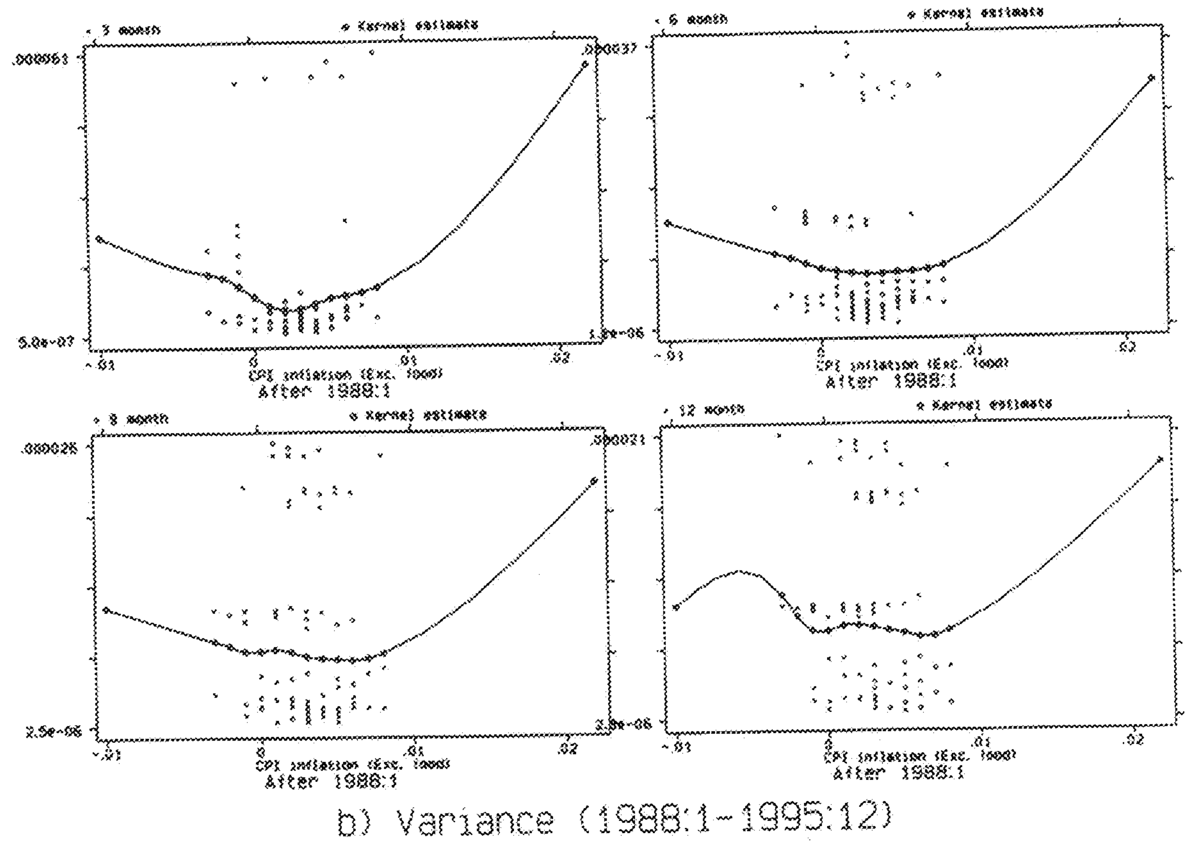


Figure 2 (cont'd): Inflation and Inflation Volatility in Canada, 1975-1988

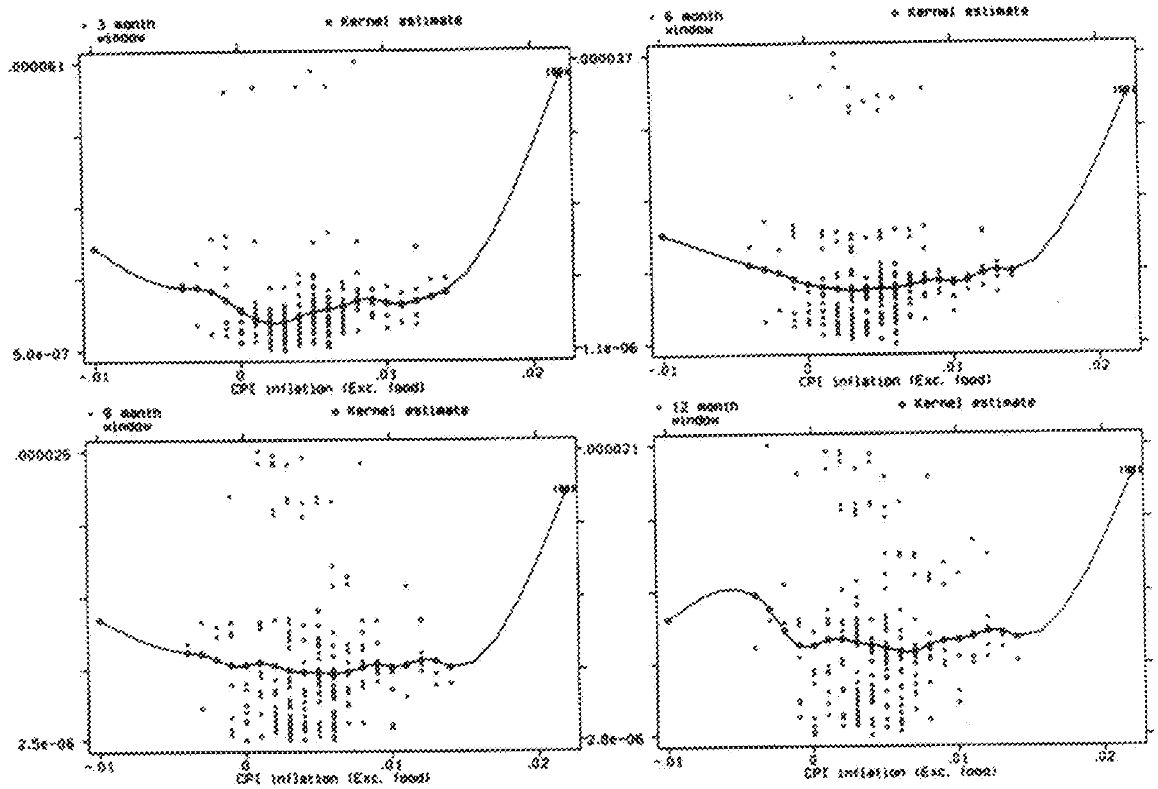


Figure 2 (cont'd): Inflation and Inflation Volatility in Canada, 1988-1995

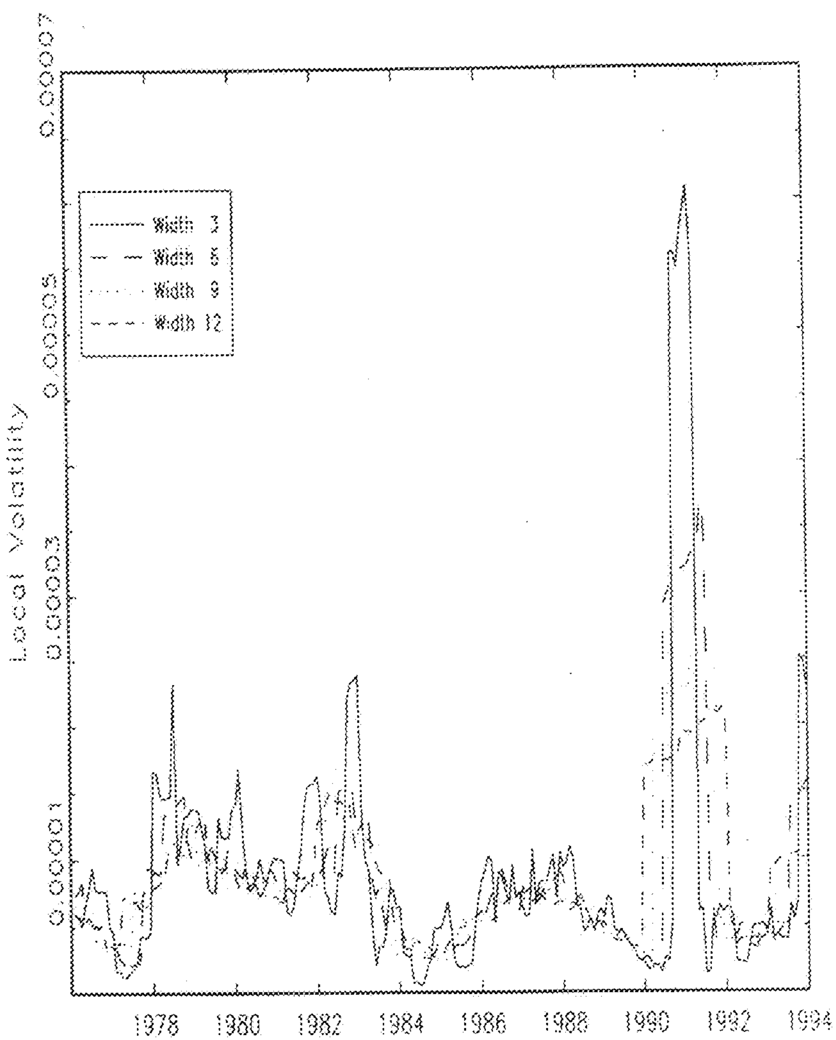


Figure 3: Local Volatility Measured by Variance

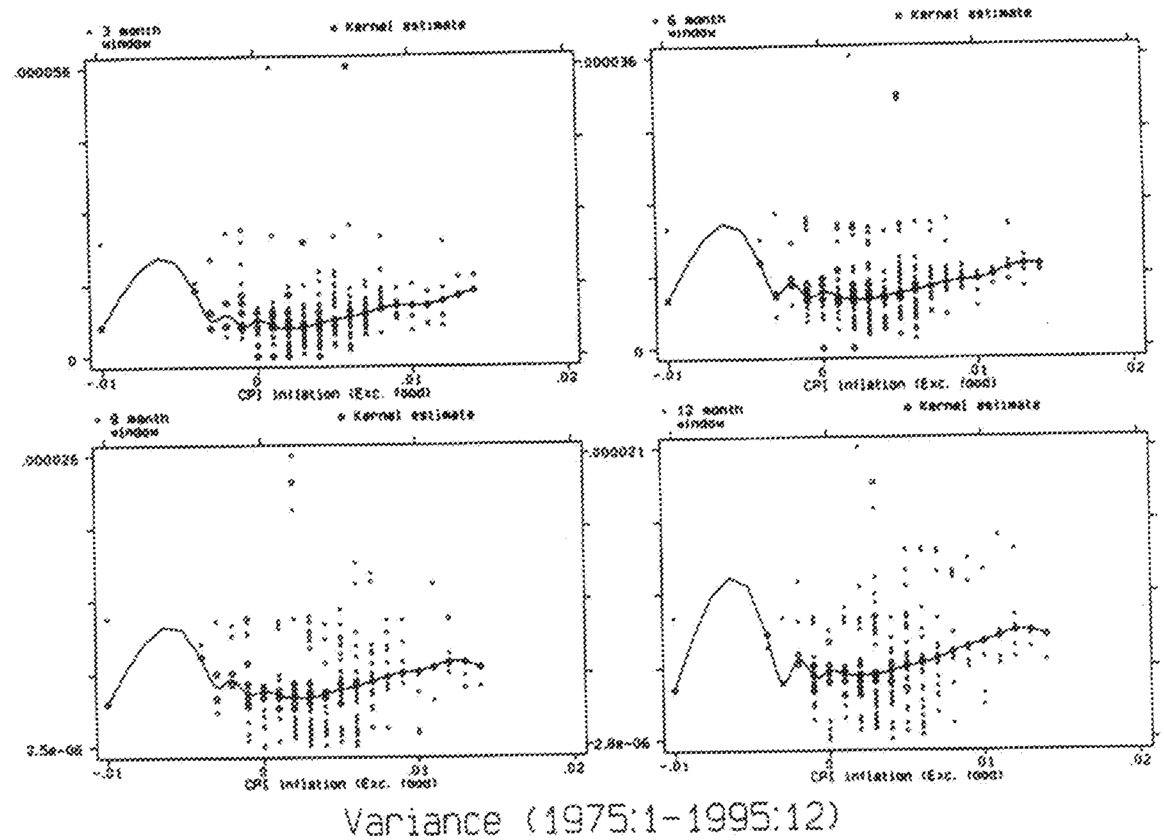


Figure 4: Inflation and Inflation Volatility with GST Dummy, 1975-1995

Appendix A: Estimates of Mean Absolute Deviation

Table 9: Estimates of the Relationship between the Inflation Rate and Its Volatility Measured by Mean Absolute Deviation

Dep. Var.: Mean Absolute Dev. of Inf.				Monthly Data: 1975:1-1995:12	
Window Width J	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\hat{\alpha}_2$	$\hat{\delta}$	R^2
3	**.0022 (.0001)	-.0381 (.0294)	**10.2030 (2.2570)	-	.1340
6	**.0021 (.0001)	-.0221 (.0203)	**5.5400 (1.5530)	-	.0870
9	**.0021 (.0001)	-.0071 (.0163)	*3.0368 (1.2470)	-	.0521
12	**.0021 (.0000)	-.0001 (.0138)	*2.1424 (1.0510)	-	.0501
3	**.0021 (.0001)	-.0169 (.0231)	**4.9630 (1.8010)	**.0037 (.0003)	.4803
6	**.0020 (.0001)	-.0047 (.0172)	**3.7025 (1.3210)	**.0016 (.0001)	.3557
9	**.0020 (.0001)	-.0035 (.0150)	2.2157 (1.9310)	**.0008 (.0001)	.2099
12	**.0020 (.0000)	-.0076 (.0133)	1.6887 (1.0110)	**.0004 (.0001)	.1331

Note: Standard errors are in parentheses. The beginning and end of the samples vary with the window width. This table reports the OLS estimates of the regression equations

$$|\omega_t|(J) = \alpha_0 + \alpha_1 \pi_t + \alpha_2 \pi_t^2 + \epsilon_t,$$

$$|\omega_t|^2(J) = \alpha_0 + \alpha_1 \pi_t + \alpha_2 \pi_t^2 + \delta\, Dummy_{Jt} + \epsilon_t,$$

where $|\omega_t|^2(J)$ is defined in equation (2), $Dummy_{Jt}$ is the dummy variable that takes value one for $t=$ 1991:1-$J,\ldots,$1991:1+J, and zero otherwise, and π_t is the inflation rate. R^2 is the coefficient of determination of the regression.

** Coefficient estimate is different from zero at the one percent level.

* Coefficient estimate is different from zero at the five percent level.

Table 10: Correlations between Interest Rate Spread and Local Volatility Measured by Mean Absolute Deviation

Monthly data: 1975:1-1995:12

	1975-1995		1975-1987		1988-1995	
Window Width, J	Corr.	*p*-value	Corr.	*p*-value	Corr.	*p*-value
3	.0569	.1864	.0400	.3126	.1304	.1081
6	.0996	.0618	.0994	.1173	.1845	.0454
9	.0406	.2679	.0312	.3575	.1387	.1103
12	.0795	.1154	.0915	.1474	.0237	.4202
Average inflation	.4490		.5632		.2645	

Note: Inflation is the log difference of monthly CPI inflation, excluding food and energy, multiplied by 100. Volatility is $|\omega|_t^2(J)$ as defined in equation (2). Since the volatility measure includes lead and lag values of inflation sample sizes vary across window widths. Sample periods are non-overlapping. *p*-values are for the test for independence. The test statistic $\sqrt{T_J}\rho(J)$, where T_J is the number of observations for window width J, and $\rho(J)$ is the estimate of the correlation coefficient, has an asymptotic standard normal distribution.

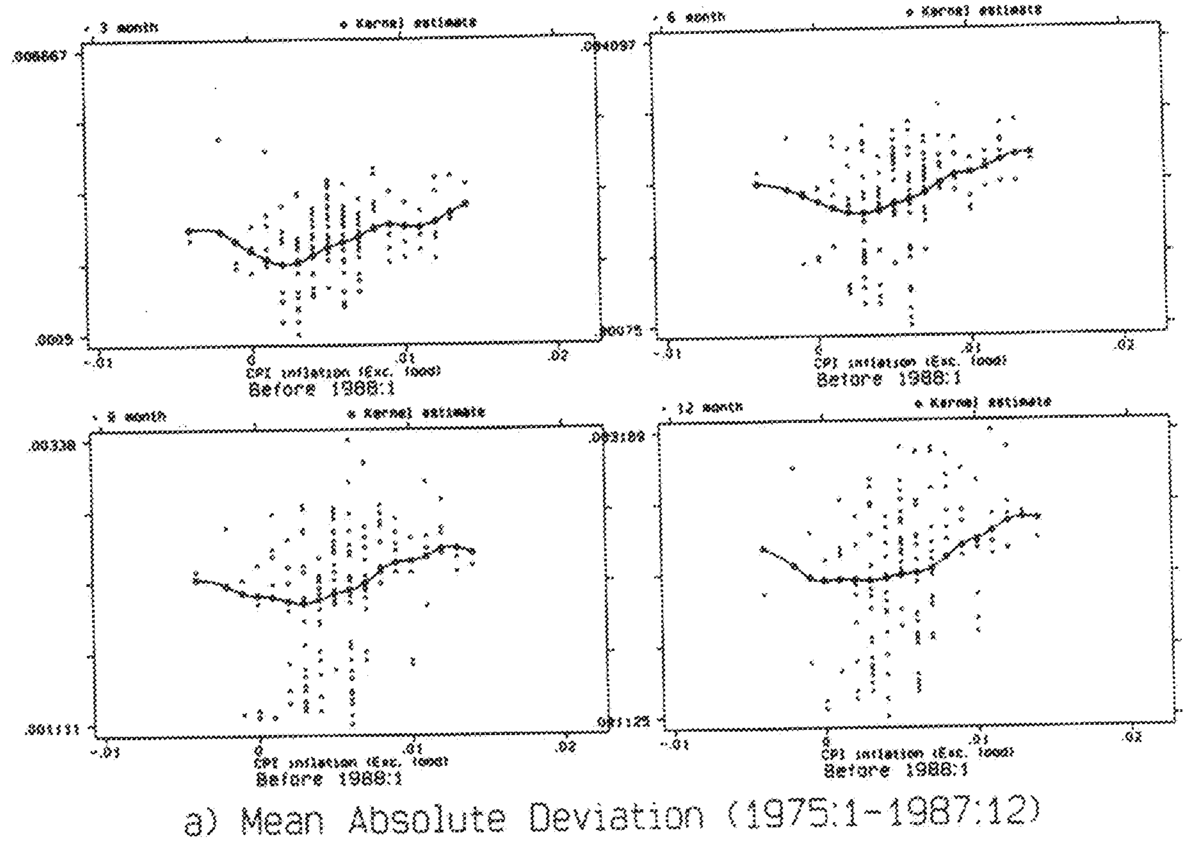
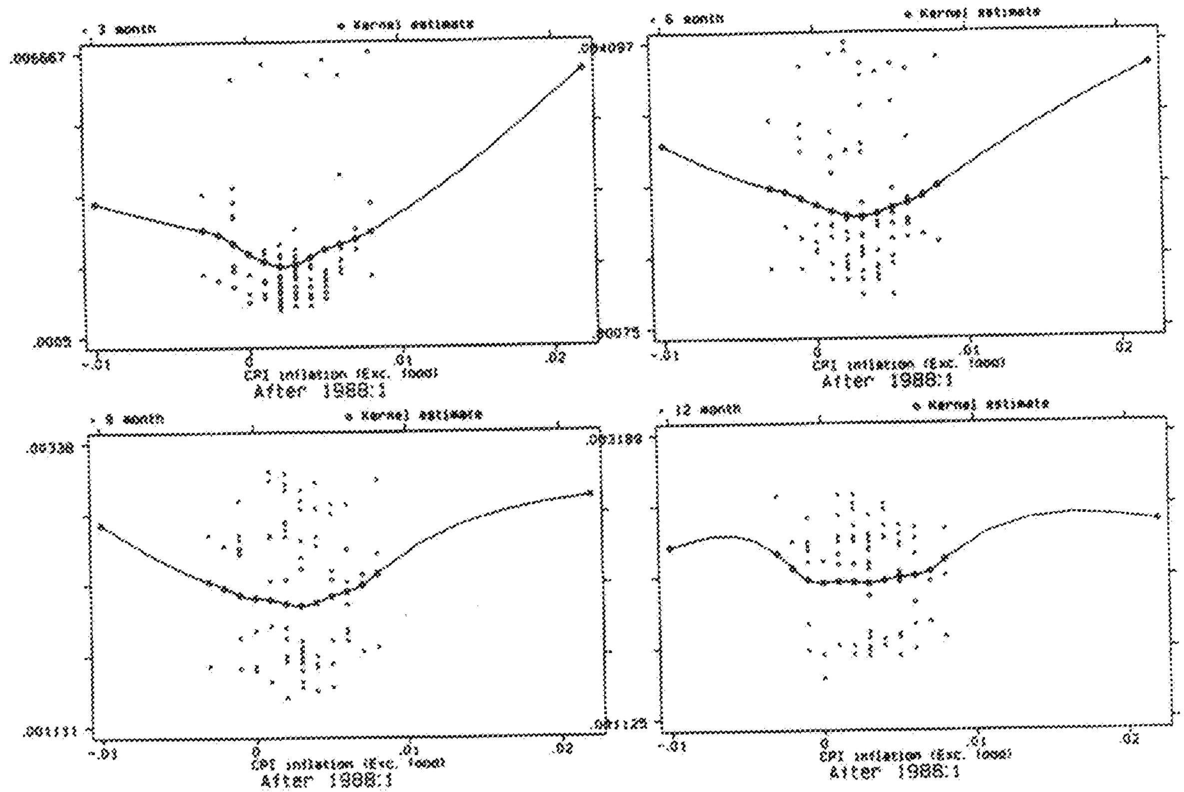
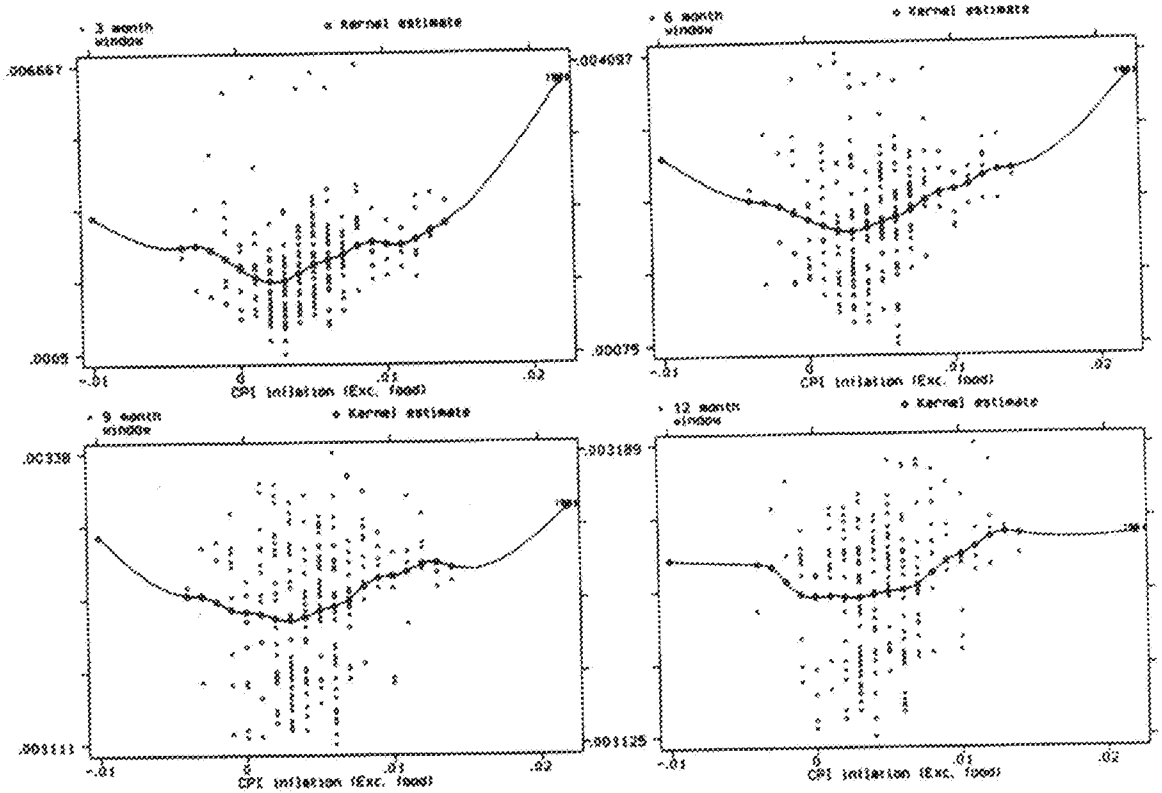


Figure 5: Inflation and Inflation Volatility in Canada, 1975-1995



b) Mean Absolute Deviation (1988:1-1995:12)

Figure 5 (cont'd): Inflation and Inflation Volatility in Canada, 1988-1995



c) Mean Absolute Deviation (1975:1-1995:12)

Figure 5 (cont'd): Inflation and Inflation Volatility in Canada, 1975-1988

Appendix B: The Kalman Filter–ARCH Model

Our Kalman filter-ARCH model is standard [see, e.g. Evans (1991)]. Inflation rate (π_t) is specified as an autoregressive process [see also equation (7) in the text]

$$\pi_{t+1} = \mathbf{x}_t \beta_{t+1} + \xi_{t+1}, \quad \xi_{t+1} \sim N(0, \Lambda_t),$$

where $\mathbf{x}_t = [1, \pi_t, \ldots, \pi_{t-k}]$ and t = 1, ..., T. We assume that the time varying state parameters evolve according to the transition equation

$$\beta_{t+1} = \beta_t + V_t, \quad V_t \sim N(0, \Gamma_t). \tag{15}$$

The specified mean inflation process can be estimated recursively. Assume that estimated parameter vector is unbiased at time $t-1$, and $\hat{\beta}_{t-1} - \beta_{t-1} \sim N(0, \Sigma_{t-1})$. Then the best predictor of β_t conditional upon all the information available at time $t-1$ is $\hat{\beta}_{t-1}$, or $\hat{\beta}_{t|t-1} = \hat{\beta}_{t-1}$, and its variance-covariance matrix evolves according to

$$\Sigma_{t|t-1} = \Sigma_{t-1} + \Gamma_t.$$

The updating equations for β_t and Σ_t are, respectively, given by,

$$\hat{\beta}_t = \hat{\beta}_{t|t-1} + \Sigma_{t|t-1}\mathbf{x}_t'\Omega_t^{-1}(y_t - \mathbf{x}_t\hat{\beta}_{t|t-1}), \tag{16}$$

and

$$\Sigma_t = \Sigma_{t|t-1} - \Sigma_{t|t-1}\mathbf{x}_t'\Omega_t^{-1}\mathbf{x}_t\Sigma_{t|t-1}, \tag{17}$$

where we defined $\Omega_t = \mathbf{x}_t\Sigma_{t|t-1}\mathbf{x}_t' + \Lambda_t$. The prediction errors are calculated by

$$v_t = y_t - \mathbf{x}_t\hat{\beta}_{t|t-1}. \tag{18}$$

In this paper, we explore if prediction errors (v_t) have ARCH(p) structures:

$$v_t = c + \xi_t, \quad \xi_t \sim N(0, \Lambda_t), \tag{19}$$

where $\Lambda_t = \gamma_0 + \sum_l^p \gamma_l \xi_{t-l}^2$.

To test for the ARCH effect the Lagrange multiplier tests are appropriate. For a more detailed discussion of the Kalman filter see Harvey (1981), and of the ARCH(p) models see Engle (1982).