# Testing first- and second-order stochastic dominance

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## I. INTRODUCTION

Tests for stochastic dominance are important tools for the comparison of distributions between pairs of random variables. In finance, dominance criteria can potentially provide an unambiguous ranking of the desirability of different assets while placing only general restrictions on the preferences of investors. In welfare economics, dominance concepts allow for the ranking of income distributions using generally accepted welfare criteria. Existing dominance test procedures suffer from a number of weaknesses.

One weakness concerns restrictions on the class of distribution functions that may be used as a basis for testing. While early literature in this area typically ignored sampling errors, this issue has been addressed in more recent literature, but only at the cost of restricting the class of parametric distributions discussed; see e.g. Stein and Pfaffenberger (1986). Since dominance criteria are attractive primarily because they allow for a ranking of returns on risky assets or income distributions, while placing only weak restrictions on preferences, it is important that dominance tests retain a degree of generality, and hence remain nonparametric in nature.

Another weakness of existing dominance tests is that these often specify the null hypothesis improperly; see e.g. Bishop et al. (1989). For example, most test procedures make use of the null hypothesis that two distribution or quantile functions are identical. The hypothesis of dominance may be viewed as an hypothesis of inequality in a particular direction between two distribution or quantile functions. If such an hypothesis is rejected, then dominance cannot be sustained, a result that may or may not be caused by the equality of the two distribution or quantile functions. On the other hand, if the null hypothesis of equality is rejected, then the two distributions cannot be said to be equal, but the cause may or may not be that one dominates the other. Thus the null hypothesis of equality is not very helpful in providing information about dominance; see Levy (1992, p. 574).

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A third weakness is that existing test procedures are not appropriate for data that exhibit weak dependence within samples and/or association between samples. This is particularly important in finance applications. For example, returns on assets are obviously determined and sampled jointly, and a positive correlation between returns will reduce the variance of the differences between distribution or quantile function estimates.

Xu et al. (1994) develops new distribution-free test procedures for stochastic dominance that address all of these weaknesses and are based on sample quantiles ( a quantile function being the inverse of the corresponding distribution function).

## II. THE TESTS

If $D = Q(X) - Q(Y)$ represents the (vector) difference between two quantile functions at $k$ points in the interval $s = [0,1]$, then $X$ dominates $Y$ in the first degree (FSD) if the null hypothesis $D \geq 0$ holds; this is tested against the alternative hypothesis $D \ngeq 0$. Second-degree stochastic dominance (SSD) implies two corresponding hypotheses based upon differences in the two cumulative quantile functions. Points on the quantile and cumulative quantile functions may be estimated freely or subject to the restraints implied by the null hypothesis.

The test for FSD is a standardized quadratic form of the differences between the free and restricted estimates of two quantile functions at $k$ points in the interval $s$. The test for SSD rests on a corresponding criterion using the cumulative quantile functions. These tests are based upon a version of the tests for equality and inequality restrictions developed by Kodde and Palm (1986) and Wolak (1989a and b). In Xu et al. (1994) $k = 20$ and the variance-covariance matrices of the distributions of estimated differences in quantile or cumulative quantile functions are estimated by the moving block bootstrap.

## III. APPLICATIONS

Two special cases are considered: when the sample observations are independently and identically distributed, the random variables being either jointly dependent or independent. The empirical application evaluates the dominance relationships between one- and two- month U.S. T-bill yields. The latter dominates the former in both the first and second degree.

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