# Inference for Generalized Gini Indices Using the Iterated-Bootstrap Method

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Inference using the iterated-bootstrap method proposed by Hall is appealing for cases in which the percentile method needs to be used but the nominal level of a confidence interval has to be adjusted. One natural application is for generalized Gini indices of income inequality. When applying these theoretical inequality measures directly to sample data for the purpose of statistical inference, economists must come up with some measure of sampling variation. This is particularly the case when the index estimates are compared over time to infer information on the changes of social welfare and inequality. Although there are difficulties in the existing inferential procedures, a more intuitive approach is to use the iterated-bootstrap method.

KEY WORDS: Generalized Gini index; Income inequality; Iterated-bootstrap; Percentile method; Bias correction.

The Gini index (or coefficient) is probably the most used measure of income inequality. It represents a special case of generalized Gini indices that allow the social planner to select a level of inequality aversion and to stress the different proportions of the income distribution. Chakravarty (1988), Donaldson and Weymark (1980), and Yitzhaki (1983) proposed two families of generalized Gini indices, the S-Gini and E-Gini indices.

To apply these indices directly to sample data, economists need to use some reliable inferential procedures. Earlier attempts, including that of Moothathu (1990), discussed the statistical inference for the Gini index assuming specific parametric income distributions. In reality, the unknown nature of the population income distributions restricts the wide applicability of this method. Although the $U$ statistics and related theory can be used to justify the asymptotic distribution of these generalized Gini-index estimators, the finite-sample properties are generally unknown and the variance estimates are usually difficult to compute (see Lee 1990; Yitzhaki 1991). Although it is possible to derive the asymptotic distribution of the index estimators based on a small number (say 20) of order statistics or Lorenz-curve ordinates of the income data, this is not desirable because of the bias introduced into the index estimators. To solve these problems, the jackknife and bootstrap methods may be used. Yitzhaki (1991) presented the jackknife method for computing variances for generalized Gini-index estimates. Generally, the jackknife variance estimator is only an approximation to the bootstrap variance estimator when the estimator is sufficiently regular. But the generalized-Gini-index estimators are functions of sample quantiles that are not regular statistics (see Shao and Tu 1995, pp. 202–203). The bootstrap method could be a better choice if the estimators are asymptotically pivotal. But the generalized-Gini-index estimators are not asymptotically pivotal. Under this circumstance, the percentile method with bias correction using Hall's (1992) iterated bootstrap is a natural choice.

The remainder of the article is organized as follows. In Section 1, the generalized-Gini-index estimators are discussed. Section 2 introduces the iterated-bootstrap method of confidence-interval estimation for comparing two generalized Gini indices. Illustrative examples are given in Section 3. Finally, some concluding remarks are offered in Section 4.

## 1. THE GENERALIZED–GINI–INDEX ESTIMATORS

Let $n$ ordered nonnegative incomes from the cdf $P(\cdot)$ [or the pdf $p(\cdot)$] of a continuous random variable $Y$ be $y_{(1)} \leq y_{(2)} \leq \cdots \leq y_{(n)}$, which form a sample $\chi$. A generalized-Gini-index estimator, say $\hat{\theta}_n$, can be viewed as a mapping $\hat{\theta}_n$: $\Re^n \to \Re^1$, or a function $\hat{\theta}_n(y_{(1)}, y_{(2)}, \cdots, y_{(n)})$. Specifically, the S-Gini (relative) index estimator is given by

$$\hat{I}_R^\delta(y) = 1 - \frac{1}{n^\delta} \sum_{i=1}^{n} \left((n-i+1)^\delta - (n-i)^\delta\right) \frac{y_{(i)}}{\hat{\mu}}, \quad (1)$$

where $\hat{\mu} = \sum_{i=1}^n y_{(i)}/n$ is the sample mean and $\delta > 1$ is an inequality-aversion parameter (see Donaldson and Weymark 1980; Yitzhaki 1983). By choosing different values for the parameter $\delta$, Equation (1) represents a family of S-Gini indices of inequality. These indices are continuous, scale free, S-concave, and invariant to arbitrary replications of the income vector; see Blackorby and Donaldson (1978) and Donaldson and Weymark (1980).

The E-Gini (relative) index estimator is given by

$$\hat{I}_R^\alpha(y) = 2\left(\frac{1}{n}\sum_{i=1}^{n}\left(\frac{1}{n} - \frac{\sum_{j=1}^{i} y_{(j)}}{n \cdot \hat{\mu}}\right)^\alpha\right)^{1/\alpha}, \quad (2)$$

where $\alpha \geq 1$ is an inequality-aversion parameter (see Chakravarty 1988). For different values of the parameter $\alpha$, Equation (2) represents a family of E-Gini indices of inequality. The properties of E-Gini indices are similar to those of S-Gini indices.


Journal of Business & Economic Statistics
April 2000, Vol. 18, No. 2

Note that two sets of indices are identical if $\delta = 2$ in Equation (1) and $\alpha = 1$ in Equation (2) because

$$\hat{I}_R^{\delta=2}(y) = 1 - \frac{1}{n^2}\sum_{i=1}^{n}\{[(n-i)+1]^2 - (n-i)^2\}\,\frac{y_{(i)}}{\hat{\mu}}$$
$$= 1 - \frac{1}{n^2}\sum_{i=1}^{n}(2n-2i+1)\,\frac{y_{(i)}}{\hat{\mu}}$$
$$= \frac{2}{n^2\hat{\mu}}\sum_{i=1}^{n} i y_{(i)} - \frac{n+1}{n},$$

and

$$\hat{I}_R^{\alpha=1}(y) = 2\left(\frac{1}{n}\sum_{i=1}^{n}\left(\frac{i}{n} - \frac{\sum_{j=1}^{i} y_{(i)}}{n\hat{\mu}}\right)\right)$$
$$= \frac{2}{n}\sum_{i=1}^{n}\frac{i}{n} - \frac{2}{n^2\hat{\mu}}\sum_{i=1}^{n}(n-i+1)y_{(i)}$$
$$= \frac{2}{n^2\hat{\mu}}\sum_{i=1}^{n} i y_{(i)} - \frac{n+1}{n}.$$

When $\delta \neq 2$ and $\alpha \neq 1$, the key difference between the two sets of indices is that they treat different data points in the distributions differently. More specifically, the S-Gini index estimator given in Equation (1) may be equivalently expressed as a function of *all* empirical Lorenz-curve ordinates:

$$\hat{I}_R^{\delta}(y) = 1 - \sum_{i=1}^{n} c_i \hat{L}\left(\frac{i}{n}\right), \tag{3}$$

where

$$\hat{L}\left(\frac{i}{n}\right) = \frac{\sum_{j=1}^{i} y_{(j)}}{\sum_{j=1}^{n} y_{(j)}} = \frac{\sum_{j=1}^{i} y_{(j)}}{n\cdot\hat{\mu}} \tag{4}$$

is the empirical Lorenz-curve ordinate evaluated at $i/n$, and $c_i'$s are defined as $c_i = ((n-i+1)^{\delta} - 2(n-i)^{\delta} + (n-i-1)^{\delta})/n^{\delta-1}$, for $i = 1, 2, \ldots, n-1$, and $c_n = 1/n^{\delta-1}$. Equation (3) shows that the S-Gini index estimator is a linear function of the empirical Lorenz-curve ordinates with weights related to the ranks of individual incomes. Similarly, the E-Gini-index estimator given in Equation (2) can be expressed as a function of *all* empirical Lorenz-curve ordinates:

$$\hat{I}_R^{\alpha}(y) = 2\left(\frac{1}{n}\sum_{i=1}^{n}\left(\frac{i}{n} - \hat{L}\left(\frac{i}{n}\right)\right)^{\alpha}\right)^{1/\alpha}. \tag{5}$$

Equation (5) shows that the E-Gini-index estimator is a nonlinear function of the empirical Lorenz-curve ordinates with weights determined by the corresponding weights of income shares.

## 2. ITERATED–BOOTSTRAP–BASED STATISTICAL INFERENCE

It is known that the S- and E-Gini indices should be estimated based on all of the $n$ data points according to Equations (1) [or (3)] and (2) [or (5)], respectively. To evaluate if the difference between two point-estimates of the S-Gini (or E-Gini) index is statistically significant, one common practice is to compute the standard deviations of the difference in point estimates and then proceed to construct a statistical test. Because of the difficulties discussed in the introduction, the bootstrap method may be considered.

The bootstrap method works best when asymptotically pivotal quantities are bootstrapped. But the generalized-Gini-index estimators are not pivotal quantities in the sense that their asymptotic distributions are functions of unknowns. As shown in Equations (3) and (5), the generalized-Gini-index estimators are functions of empirical Lorenz-curve ordinates, the asymptotic variance–covariance matrix of which is a function of unknowns (Beach and Davidson 1983). Hence, the generalized-Gini-index estimators are not asymptotic pivotal.

When the generalized-Gini-index estimators do not have analytical scale (or standard deviation) estimates and are not asymptotically pivotal, I can use the percentile method with bias correction or adjusting the nominal level of confidence-intervals via iterated bootstrap. In practice, the double bootstrap is sufficient and hence it is a special form of the iterated bootstrap. Bootstrap iteration can substantially improve the estimation of confidence intervals. According to Hall (1992), a significant part of the future of bootstrap methods lies in this direction, using techniques such as the iterated bootstrap to marry the best features of different methods and thereby obtaining new techniques with widespread applicability.

The key to the iterated-bootstrap method is to find a tuning parameter $t = t(n)$ for the nominal level $a$ of a confidence interval for a true parameter $\theta$ given the parameter estimate $\hat{\theta}_n$ from the original sample $\chi$. Under the assumption of $t(n) \to 0$ as $n \to \infty$, the tuning parameter $t$ may be chosen to be an additive correction to the nominal level $a$ and be chosen such that either

$$P\{\theta \in (-\infty, \hat{x}_{a+t})\} = a \tag{6}$$

or

$$P\{\theta \in (\hat{x}_{(1-a-t)/2}, \hat{x}_{(1+a+t)/2})\} = a, \tag{7}$$

where $\hat{x}$ is a critical value. For this purpose, I shall focus on equal-tailed, two-sided confidence intervals. Let the nominal level be $a$ such that $(1/2) < a < 1$. Then $(\hat{x}_{1-\xi}, \hat{x}_{\xi})$ is the nominal $a$-level confidence interval for $\theta$ with $\xi = (1/2)(1+a)$. The coverage error is at most of order $n^{-1}$; that is,

$$P\{\theta \in (\hat{x}_{1-\xi}, \hat{x}_{\xi})\} = a + O(n^{-1}). \tag{8}$$

Please note that $t$ is a function of $n$, and it may be expressed as an asymptotic power series in $n^{-1/2}$ in the case of correcting a one-sided interval and an asymptotic series in $n^{-1}$ in the case of correcting a two-sided interval. If $a$ is adjusted to the new level $a+t, \xi$ should be changed to

$\xi + (1/2)t$. It is possible and desirable to select $t$ so that

$$P\{\theta \in (\hat{x}_{1-\xi-(t/2)}, \hat{x}_{\xi+(t/2)})\} = a. \quad (9)$$

To estimate $t$ in bootstrap iteration, let $\chi^{**}$ denote a resample drawn randomly, with replacement, from a resample $\chi^*$, which is again a resample from the original sample $\chi$. Let $\hat{\theta}^{**}$ denote the version of $\hat{\theta}_n$ computed from $\chi^{**}$ and $\hat{\theta}^*$ denote the version of $\hat{\theta}_n$ computed from $\chi^*$. Let $\hat{x}^*$ be the critical value computed for the empirical distribution of $\hat{\theta}^*$ based on $\chi$. Then

$$\hat{\pi}(t) = P\{\hat{\theta}_n \in (\hat{x}^*_{1-\xi-(t/2)}, \hat{x}^*_{\xi+(t/2)})|\chi\} \quad (10)$$

is an estimate of

$$\pi(t) = P\{\theta \in (\hat{x}_{1-\xi-(t/2)}, \hat{x}_{\xi+(t/2)})\}. \quad (11)$$

The solution $\hat{t}$ of $\hat{\pi}(t) = a$ is an estimate of the solution of $\pi(t) = a$. The iterated-bootstrap interval is

$$\mathcal{I} = (\hat{x}_{1-\xi-(\hat{t}/2)}, \hat{x}_{\xi+(\hat{t}/2)}). \quad (12)$$

According to Hall (1992), $\hat{t}$ can be computed as follows: (1) Draw $B_1$ independent resamples $\chi_1^*, \ldots, \chi_{B_1}^*$ by sampling randomly, with replacement, from the original sample $\chi$; (2) for each $i$ $(1 \leq i \leq B_1)$, draw $B_2$ independent resamples $\chi_{i1}^{**}, \ldots, \chi_{iB_2}^{**}$ by sampling randomly, with replacement, from $\chi_i^*$. Use $\hat{\theta}_i^*$ and $\hat{\theta}_{ij}^{**}$ for values of $\hat{\theta}_n$ from $\chi_i^*$ and $\chi_{ij}^{**}$, respectively, instead of $\chi$; (3) rank $\{\hat{\theta}_i^*, 1 \leq i \leq B_1\}$ as $\hat{\theta}^*_{(1)} \leq \cdots \leq \hat{\theta}^*_{(B_1)}$, and rank $\{\hat{\theta}_{ij}^{**}, 1 \leq j \leq B_2\}$ as $\hat{\theta}^{**}_{i,(1)} \leq \cdots \leq \hat{\theta}^{**}_{i,(B_2)}$ for every $i$; (4) let $v_k(b)$, for $k = 1, 2$, be monotone functions taking values in the sequence $1, \ldots, B_k$ and such that $v_k(b)/B_k \simeq b$ for $k = 1, 2$ and $0 \leq b \leq 1$; (5) the $\hat{\theta}^*_{(v_1(b))}$ is an approximation to $\hat{x}_b$ and

$$\hat{\pi}_{B_1B_2}(t) = B_1^{-1} \sum_{i=1}^{B_1} I(\hat{\theta}_n \in (\hat{\theta}^{**}_{i,(v_2(1-\xi-(t/2)))}, \hat{\theta}^{**}_{i,(v_2(\xi+(t/2)))})), \quad (13)$$

with $I(\cdot)$ being an indicator function, is an approximation to $\hat{\pi}(t)$. Choose $\hat{t}_{B_1B_2}$ to solve the equation

$$\hat{\pi}_{B_1B_2}(t) = a \quad (14)$$

as nearly as possible. Then the confidence interval

$$\mathcal{I}_{B_1B_2} = (\hat{\theta}^*_{(v_1(1-\xi-(\hat{t}_{B_1B_2/2})))}, \hat{\theta}^*_{(v_1(\xi+(\hat{t}_{B_1B_2/2})))}) \quad (15)$$

is a computable approximation to the interval $\mathcal{I}$ defined in Equation (12).

For the purpose of computation, it is useful to note the following: Given $1/2 < a < 1$ and $\xi = (1/2)(1 + a)$, $[\hat{x}_{1-\xi}, \hat{x}_\xi]$ represents the nominal $a$-level confidence interval. One may adjust $1 - \xi$ and $\xi$ to $1 - \xi - (t/2)$ and $\xi + (t/2)$, respectively. To set the range for $t$, note that, when $t = 2(1 - \xi), 1 - \xi - (t/2) = 0$ and $\xi + (t/2) = 1$ and that, when $t = -2(1 - \xi), 1 - \xi - (t/2) = 2(1 - \xi)$ and $\xi + (t/2) = 2\xi - 1$.

As $v_1, v_2, B_1$, and $B_2$ diverge in such a way that $v_i(b)/B_i \to b$, for $i = 1, 2$ and $0 \leq b \leq 1$, I have $\hat{\pi}_{B_1B_2} \to \hat{\pi}$ and $\hat{\theta}^*_{(v_1(b))} \to \hat{x}_b$ with probability 1 conditional on $\chi$. This ensures that $\mathcal{I}_{B_1B_2} \to \mathcal{I}$ as $B_1B_2 \to \infty$, at least within the limitations imposed by discreteness in the definition of the latter interval. Booth and Hall (1994) noted that $B_1B_2$ should be at least $10^5$, $B_2$ should be considerably smaller than $B_1$, and it is optimal to set $B_2$ to be approximately a constant multiple of $B_1^{1/2}$.

To compare two generalized Gini indices, the iterated-bootstrap method can be used to compute the confidence interval of the difference between the two indices as in Equation (15). If the confidence interval does not contain 0, it suggests that the difference is statistically significant at a certain level; otherwise, the difference is not statistically significant.

## 3. ILLUSTRATIVE EXAMPLES

In their survey, Levy and Murnane (1992) reported the Gini-index estimates from 1967 to 1986 (see Levy and Murnane 1992, table 2: part 1). Based on the point-estimates of the Gini index computed from the sample, they concluded that earning inequality did not change much in the 1970s but increased rapidly in the 1980s. Two issues that are potentially important were not addressed so that the conclusion would be statistically sound: (1) Will the conclusion be affected if the sampling variation inherent in the sample estimates are duly considered? (2) Will the conclusion be affected if inequality-aversion changes to reflect some different social norms? To address these two issues, I adopt the S- and E-Gini-index estimators that allow changes of the inequality-aversion parameter, with the iterated-bootstrap method for statistical inference. To evaluate the changes of income inequality of the United States over time, I apply the proposed method to the 1969, 1979, and 1988 nominal individual income data from the Panel Study of Income Dynamics. Because the S- and E-Gini indices are more general than the regular Gini index, I can use these indices to examine the changes of income inequality over time while allowing for different degrees of inequality aversion.

Table 1 shows the means and standard deviations of individual incomes, and the S- and E-Gini-index estimates and their bootstrap standard deviations for 1969, 1979, and 1988. The table shows that the means (standard deviations) of individual incomes are \$26,596.40 (\$19,437.97) in 1969, \$28,931.59 (\$24,098.20) in 1979, and \$30,816.98 (\$32,224.66) in 1988. To evaluate income inequality according to different levels of inequality aversion, I compute the S- and E-Gini indices based on the sample data.

I let the values of $\delta$ ($\alpha$) be 1.25, 2.00, and 3.50 (1.00, 4.00, and 10.00), respectively, to give different degrees of emphasis on the undesirability of inequality. When the inequality-aversion parameter $\delta$ is set low—that is, $\delta = 1.25$—the S-Gini-index estimates are .1412 in 1969, .1483 in 1979, and .1701 in 1988. Similarly, when the inequality-aversion parameter $\alpha$ is set low—that is, $\alpha = 1.00$—the E-Gini-index estimates are .3564 in 1969, .3696 in 1979, and .4074 in

Table 1. Estimated S- and E-Gini Indices, U.S. Income

| Year | Mean $ | S-Gini $\delta = 1.25$ | Index $\delta = 2.00$ | Estimates $\delta = 3.50$ | E-Gini $\alpha = 1.00$ | Index $\alpha = 4.00$ | Estimates $\alpha = 10.00$ |
|---|---|---|---|---|---|---|---|
| 1969 | 26,596.40 | .1412 | .3564 | .5314 | .3564 | .4135 | .4490 |
| | (19,473.97) | (.0039) | (.0066) | (.0067) | (.0062) | (.0067) | (.0073) |
| 1979 | 28,931.59 | .1483 | .3696 | .5484 | .3696 | .4291 | .4661 |
| | (24,098.20) | (.0046) | (.0057) | (.0053) | (.0059) | (.0059) | (.0060) |
| 1988 | 30,816.98 | .1701 | .4074 | .5901 | .4074 | .4722 | .5125 |
| | (32,224.66) | (.0055) | (.0057) | (.0053) | (.0060) | (.0063) | (.0068) |

NOTE: The sample sizes are 2,206, 4,173, and 6,244 for the years of 1969, 1979, and 1988, respectively. The number of iterations for bootstrap estimation is chosen to be 500. The numbers in the parentheses are standard deviations for mean income and bootstrap standard deviations for S- and E-Gini indices.

1988. It appears that the estimates of the S- and E-Gini indices have increased from 1969 to 1988. The changes in both indices from 1969 to 1979 are relatively small, whereas the changes from 1979 to 1988 are relatively large. When the income inequality-aversion parameter $\delta$ ($\alpha$) is set to 2.00 (4.00), the S-Gini (E-Gini) index estimates are .3564 (.4135) in 1969, .3696 (.4291) in 1979, and .4074 (.4722) in 1988. It appears that the estimates of the S- and E-Gini indices have increased. Similar observation can be made for the index estimates when the inequality-aversion parameter $\delta$ ($\alpha$) is set to 3.50 (10.00). Once again, I note that the S- and E-Gini indices are identical when $\delta = 2$ and $\alpha = 1$.

Although I have observed increases in these index estimates over time, I do not know if the increases are due to sampling variation or to changes in the population income distributions. I can use the proposed inferential procedure to verify the statistical significance of the increases in the S- and E-Gini indices over time. Table 2 shows both estimates of the S- and E-Gini indices and the 95% confidence intervals of their differences between 1969 and 1979, and 1979 and 1988. Here I use $B_1 = 999$ and $B_2 = 100$ in the iterated bootstrap. As can be seen from the table, the increases in income inequality from 1969 to 1979 are very marginal even if one considers a wide range of values for the inequality-aversion parameters ($\delta$ and $\alpha$) because all of the 95% confidence intervals contain the point of 0. It is no longer the case, however, for the increases of income inequality from 1979 to 1988. Almost all of the 95% confidence intervals of the changes in inequality measures from 1978 to 1988 do not contain 0 except for the case of the S-Gini index when the inequality-aversion parameter $\delta$ is set low at 1.25. These statistical inferences confirm the observation made by Levy and Murnane (1992) based only on the point estimates of the Gini index: The income inequality did not change in the 1970s and increased rapidly in the 1980s. Although the S- and E-Gini-index estimates are higher in value and these values diverge more from each other as the values of the parameters are set higher ($\delta = 2.00$ and 3.50 and $\alpha = 4.00$ and 10.00), the statistical inferences still

Table 2. The 95% Confidence Intervals for the Differences

| Years | 1969 | | 1979 | | 1988 |
|---|---|---|---|---|---|
| S-Gini, $\delta = 1.25$ | .1412 | | .1483 | | .1701 |
| 95% conf. interval for difference | | [−.02475, .01057] | | [−.04643, .00109] | |
| S-Gini, $\delta = 2.00$ | .3564 | | .3696 | | .4074 |
| 95% conf. interval for difference | | [−.03919, .01133] | | [−.06092, −.01745] | |
| S-Gini, $\delta = 3.50$ | .5314 | | .5484 | | .5901 |
| 95% conf. interval for difference | | [−.04798, .01516] | | [−.06618, −.02069] | |
| E-Gini, $\alpha = 1.00$ | .3564 | | .3696 | | .4074 |
| 95% conf. interval for difference | | [−.03889, .01431] | | [−.06151, −.00994] | |
| E-Gini, $\alpha = 4.00$ | .4135 | | .4291 | | .4722 |
| 95% conf. interval for difference | | [−.04993, .01720] | | [−.07175, −.00277] | |
| E-Gini, $\alpha = 10.00$ | .4490 | | .4661 | | .5124 |
| 95% conf. interval for difference | | [−.05337, .01424] | | [−.07857, −.01540] | |

NOTE: The 95% confidence intervals are estimated based on the iterated-bootstrap method with $B_1 = 999$ and $B_2 = 100$. The confidence intervals provide the information on the significance of the difference between two Gini indices. When the confidence interval includes (excludes) 0, it indicates that the difference is insignificant (significant). The increases between the 1969 and 1979 generalized Gini indices are insignificant, but the increases between 1979 and 1988 indices are significant except for the increase of the S-Gini index in 1979 and 1988 when $\delta = 1.25$.

confirm the conclusion that income inequality deteriorated from 1979 to 1988 but not from 1969 to 1979.

## 4. CONCLUDING REMARKS

This article proposes the iterated-bootstrap method and applies it in analyzing the changes in income inequality over time based on sample data. This proposed method does not alter the theoretical indices for the purpose of statistical inference. It does not require estimation of the standard deviation. This method is essentially the percentile method with bias correction using the iterated bootstrap. This computation-intensive method can be applied to a wide range of cases in which meaningful theoretical quantities defined for the population can be computed only based on sample data. The percentile method with bias correction allows us to take sampling variation into full consideration in statistical inference. As an example, I use the iterated-bootstrap method to verify the statistical significance of the changes of income inequality during the 1970s and 1980s.

## ACKNOWLEDGMENTS

I have benefited a great deal from communication with Peter Hall at Australian National University, David Donaldson at University of British Columbia, and participants at the Canadian Economics Association 32nd Annual Meeting. I thank Essie Maasoumi at the Southern Methodist University for kindly providing the data. All of the remaining errors are, of course, my own.

*[Received November 1997. Revised March 1999.]*

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