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INCOME UNCERTAINTY, SUBSTITUTION EFFECT AND RELATIVE YIELD SPREADS

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The persistent downward-sloping term structure of relative yield spreads of U.S. taxable and tax-exempt bonds has been a puzzle in economics and finance. Since the late 1980s, a number of researchers have found this "anomaly" and analyzed this phenomenon from various vantage points such as market segmentation, special trading behaviors, and risk-aversion. This paper suggests that the term structure may be affected by the dynamic portfolio choice of risk-averse investors who face uncertain future income and tax rates. This paper proposes a dynamic portfolio model that can incorporate future uncertainty into a dynamic decision process. It is hoped that this model could offer some new insight into possible causes.

The term structure of relative yield spreads between U.S. taxable and tax-exempt bonds is often discussed in economics and finance (Stiglitz, 1988; Van Horne, 1994). It is commonly recognized that the tax rate of the marginal investor affects the pricing of taxable and tax-exempt bonds. There is, however, little agreement on how the tax rate as one form of market frictions works, and why relative yield spreads of the two types of long-term bonds are generally much smaller than, but occasionally greater than that of their short-term counterparts.

The theoretical attempts to study relative yield spreads of taxable and tax-exempt bonds started from Miller (1977) and Fama (1977). They noted that the corporatecapital-structure and bank tax-arbitrage could affect current relative yield spreads. Having noticed the inaccurate predictions and difficulties in reconciling the "implied tax rate" (the relative yield spread) with the actual average tax rate faced by nonbank corporations and banks, Trczinka (1982) tackled the issue by incorporating a timevarying default risk premium for prime-grade municipal bonds into his model, while Buser and Hess (1986) extended the Miller model by including financing costs. Skelton (1983), however, employed the historical restrictions of Regulation Q to explain the yield spreads.

The Tax Reform Act of 1986 had a profound impact on the tax-exempt bond market. Although relative yield spreads have become consistently narrower since then (see Table 1), the downward-sloping pattern of the term structure appeared

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Year	rýs1	rýs5	rýs 10	rýs 20	rýs 30
1953	.33202	.37085	.33271	.22707	.20681
1954	.26822	.38441	.38714	.22408	.16512
1955	.41426	.40642	.34104	.24387	.19890
1956	.41305	.27887	.27887	.21914	.18406
1957	.38013	.29398	.22178	.15101	.12198
1958	.41245	.32647	.26244	.18936	.14843
1959	.43309	.37640	.28971	.22237	.18499
1960	.42201	.36421	.29179	.22678	.16979
1961	.47558	.39681	.28376	.19023	.15052
1962	.46774	.42310	.35300	.25495	.20567
963	.46487	.41234	.35108	.26094	.21851
1964	.44143	.37507	.32716	.26492	.22073
965	.42567	.34608	.31624	.26378	.21874
1966	.33647	.31593	.26608	.22708	.20676
1967	.36952	.32472	.28602	.23880	.20808
1968	.40533	.32821	.28210	.22585	.18391
969	.34413	.27817	.21361	.13255	.08724
970	.38654	.35275	.25915	.10124	.06041
971	.40482	.39035	.29215	.13026	.07386
972	.43495	.38774	.33077	.15365	.10566
.973	.45279	.37432	.33867	.28219	.25528
974	.42640	.36592	.29954	.28220	.26236
975	.40824	.36440	.26593	.21735	.19732
976	.47469	.42438	.36061	.27580	.24239
977	.50715	.43888	.40591	.32190	.29002
978	.49281	.43615	.40846	.34620	.31776
979	.49673	.42904	.41665	.34843	.32831
980	.48539	.43923	.39986	.30800	.27467
981	.46317	.39434	.32343	.22935	.19589
982	.42365	.33616	.24884	.15360	.12476
983	.44545	.37242	.28085	.20614	.18583
984	.44031	.38338	.30016	.22236	.21006
985	.39749	.32298	.24805	.19638	.18351
986	.32539	.25131	.18076	.14832	.10816
.987	.33132	.29102	.23050	.16768	.13095
988	.27263	.28563	.24418	.17436	.15416
989	.28749	.24762	.21491	.17685	.15792
990	.25995	.25123	.22148	.19008	.17524
991	.21016	.24802	.22765	.17476	.16720
1992	.22514	.22134	.19199	.14934	.17126
993	.24251	.19841	.18629	.12630	.16155
.994	.29881	.27337	.24006	.16010	.16560

Table 1. Annual Averages of the Monthly Relative Yield Spreads: 1953-1994

Note: The annual averages of monthly relative yield spreads with term-to-maturity $i(\hat{rys}_i, i = 1, 5, 10, 20, and 30)$ from 1953 to 1994 are computed from the monthly data in *Analytical Record of Yields and Yield Spreads* published by Salomon Brothers Inc. The federal government bond yields are taxable. The prime grade municipal bond yields are tax-exempt. The relative yield spread of taxable and tax-exempt bonds for a particular term-to-maturity is defined as the ratio of the yield spread of the two types of bonds of that term-to-maturity to the yield of the corresponding taxable bond.

unchanged in general. In the 1980s, the municipal (tax-exempt) bonds became increasingly popular investment instruments among individual investors and mutual fund management firms.

Since then the research focus gradually shifted to the term structure of relative vield spreads. The relative vield spread of taxable and tax-exempt bonds for a particular term-to-maturity is defined as the ratio of the yield spread of the two types of bonds of that term-to-maturity to the yield of the corresponding taxable bond. Kochin and Park (1988) identified the downward-sloping pattern and suggested that it provides arbitrage opportunities (see Table 1). Other researchers, such as Piros (1987), Mitchell and McDade (1992), Green (1993), and Kryzanowski, Xu, and Zhang (1995), attempted to explain the factors that may cause the downward-sloping term structure. Piros (1987) noted that the yields of tax-exempt bonds must be sufficiently high to attract risk-averse investors whose future income is uncertain. By examining the behavior of property and liability insurance companies in the tax-exempt bond market, Mitchell and McDade (1992) found that the downward-sloping term structure of relative yield spreads may result from the market segmentation. Green (1993) suggested that the downward-sloping pattern is caused by the investor's taxable bond trading strategy. Kryzanowski, Xu, and Zhang (1995) found that, in addition to the time-varying risk premium and forward tax rate, the tax timing option and expected future inflation could also cause the term structure downward-sloping.

Researchers who have studied taxable versus tax-exempt bond yields have focused rather narrowly on one or another factor that may account for the observed pattern of relative yield spreads. None of these papers has considered the yield spread effect of investors who choose taxable and tax-exempt bonds in a multi-period context that includes uncertainty about future income and tax rates. This paper attempts to do exactly that based on Lucas (1978), Piros (1987), and Sargent (1987). The paper is limited, however, in that it makes no attempt to incorporate the empirical findings of other researchers, and that it assumes that the investors are not constrained from rearranging their portfolios freely. Some investors, in particular those institutional investors in these markets often lack complete flexibility to rearrange their portfolios. Their presence in the taxable and tax-exempt bond markets could be an important force affecting the term structure.¹

The rest of the paper is organized as follows. In Section I, the proposed model is introduced and discussed. Section II contains the theoretical analysis and predictions. Finally, some concluding remarks are offered in Section III.

I. MODEL

This section establishes a model that describes the risk-averse investor's dynamic portfolio choice in a nominal framework. It is different from the conventional asset pricing model (Lucas, 1978; Sargent, 1987) in that income, assets, returns, and tax payments are all nominal. Within the model framework, this paper shows how uncertain future income and tax rates, and income and substitution effects are related to the term structure of relative yield spreads.

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In this model, the representative investor is assumed to be rational and riskaverse. The investor receives income, y_t , and makes a consumption decision, c_t , and an investment decision at time t. Exogenous uncertainty is characterized by a time-homogeneous Markov process $\{X_t\}$, where X_t is the state at time t. The income process $\{y_t\}$ can be expressed as a function of $\{X_t\}$. Thus, the source of uncertainty is from future income and the uncertainty is resolved only when income becomes known.

The time-*t* traded short- and long-term securities are default-risk free, taxable one- and two-period bonds, denoted as L_{1i}^{τ} and L_{2i}^{τ} respectively. Their tax-exempt counterparts are denoted as L_{1i} and L_{2i} respectively.² These securities are denominated in units of time-*t* income. The after-tax gross returns on these securities, R_{ii}^{τ} and R_{ii} (i = 1, 2), are the extra resources available to the investor when the bonds mature. The pre-tax net returns on taxable and tax-exempt bonds, r_{ii}^{τ} and r_{ii} (i = 1, 2), are known when the bonds are purchased at time *t*. The tax rates imposed on pre-tax net returns at time t+1 and t+2 are denoted as τ_{i+1} and τ_{i+2} , respectively. The after-tax gross returns and pre-tax net returns are related by $R_{ii}^{\tau} = 1 + (1 - \tau_{i+i})r_{ii}^{\tau}$ and $R_{ii} = 1 + r_{ii}$ for i = 1, 2.

The government collects taxes from both income and yields. The tax rate imposed on income at time t, τ_p is a monotonic increasing function of the level of income and predictable with respect to the information set at time t, i.e., $\tau_t = f(y_t(X_d))$, where $\tau_t \in (0, 1)$. The same tax rate, τ_p is also the one imposed on the yield of one-period taxable bonds purchased at time t-1 because the taxable yield is paid at time t. The tax rate, imposed as of time t+1 on the yield of two-period bonds purchased at time t-1, is uncertain because the taxable yield is paid at time t+1, when the state is still unknown. This tax rate is denoted as $\tau_{H1} = f(y_{H1}(X_{H1}))$, where $\tau_{H1} \in (0, 1)$. Clearly, the uncertainty of the future tax rate implies the possibility of changes in the tax rate applicable to the investor due to possible changes in future income. Thus, the focus will be on the investor's expectations of uncertain future income and tax rates. Since uncertainty is characterized by a time-homogeneous Markov process, we specify $E_t(\tau_{H1}) = \tau_p$ i = 1, 2; that is, the expectations of the future tax rate based on the information set available at the current time is simply the current tax rate. The investor's optimization problem is constrained by

$$c_{t} + L_{1t} + L_{1t}^{\tau} + L_{2t} + L_{2t}^{\tau} \leq y_{t}(1 - \tau_{t}) + L_{1 \leftarrow 1}R_{1 \leftarrow 1} + L_{1 \leftarrow 1}^{\tau}R_{1 \leftarrow 1}^{\tau} + L_{2 \leftarrow 2}R_{2 \leftarrow 2} + L_{2 \leftarrow 2}^{\tau}R_{2 \leftarrow 2}^{\tau}$$
(1)

where $t = 0, 1, ...; y_0$ and L_0 are initial values of income and assets, respectively. Since at time tonly τ_t applies to the budget constraint, τ_t does not introduce any uncertainty at time t. This budget constraint suggests that the sum of consumption and new investment in bonds should be less than or equal to the sum of after-tax income and assets available for reinvestment.

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The investor has a time-separable, strictly concave, monotonic and twice differentiable utility function, u = u(c). The discount rate is defined as β which is less than one and greater than zero. The set of choice variables at time t is defined by $A_t = \{c_b, L_{1b}, L_{2b}, L_{1b}^{\tau}, L_{2d}^{\tau}\}$. Then the individual investor solves the following dynamic portfolio choice problem

$$\max_{\{A_i\}} E_0 \sum_{i=0}^{\infty} \beta^i u(c_i)$$
⁽²⁾

subject to Equation 1. At time t, the variables $y_b \tau_b d_b R_{1b} \tau_{1b}^* R_{2b}$ and τ_{2t}^* are known, conditional on the information set available at time t.

II. THEORETICAL PREDICTIONS

This section shows how the term structure of relative yield spreads and uncertainty introduced by the exogenously-given income process are related.

Solve the problem by forming the Lagrangian function,

$$J = E_0 \sum_{i=0}^{\infty} \beta^{i} \{ u(c_i) + \lambda_i [y_i (1 - \tau_i) + L_{1 \leftarrow 1} R_{1 \leftarrow 1} + L_{1 \leftarrow 1}^{\tau} R_{1 \leftarrow 1}^{\tau} + L_{2 \leftarrow 2} R_{2 \leftarrow 2} + L_{2 \leftarrow 2}^{\tau} R_{2 \leftarrow 2}^{\tau} - c_i - L_{1 \leftarrow 1} - L_{1 \leftarrow 1}^{\tau} - L_{2 \leftarrow 1} - L_{2 \leftarrow 1}^{\tau}] \},$$
(3)

where $\{\lambda_{i}\}$ is a sequence of random Lagrange multipliers. Given that the solution exists for this Markov model,³ some of the first-order conditions are

$$u'(c_t) - \lambda_t = 0, \tag{4}$$

$$-\lambda_t + \beta E_t(\lambda_{t+1}R_1) = 0, \qquad (5)$$

$$-\lambda_t + \beta E_t (\lambda_{t+1} R_{1t}^{\tau}) = 0, \qquad (6)$$

$$-\lambda_t + \beta^2 E_t (\lambda_{t+2} R_{2t}) = 0, \tag{7}$$

and

$$-\lambda_t + \beta^2 E_t(\lambda_{t+2} R_{2t}^{\tau}) = 0. \tag{8}$$

Rearranging these conditions yields

$$u'(c_t) = \beta E_t[u'(c_{t+1})R_{1t}], \tag{9}$$

$$u'(c_t) = \beta E_t[u'(c_{t+1})R_{1t}^{\tau}], \qquad (10)$$

$$u'(c_t) = \beta^2 E_t[u'(c_{t+2})R_{2t}], \tag{11}$$

and

$$u'(c_t) = \beta^2 E_t [u'(c_{t+2}) R_{2t}^{\tau}].$$
(12)

From Equations 9 and 10 with the time subscripts shifted backward once and the definition of conditional covariance, it can be shown

$$r_{1 \leftarrow 1} - r_{1 \leftarrow 1}^{\mathfrak{r}} \left[1 - E_{t}(\tau_{t}) \right] = \operatorname{Cov}_{t} \left[u'(c_{t}), r_{1 \leftarrow 1}^{\mathfrak{r}} (1 - \tau_{t}) \right] \left[E_{t} u'(c_{t}) \right]^{-1}.$$
(13)

Since $E_t(\tau_t) = \tau_t$ and there is no uncertainty in both income and consumption at time t, the right-hand side of Equation 13 is zero, yielding the (indifference) tax rate

$$(r_{1t-1}^{x} - r_{1t-1}) / r_{1t-1}^{x} = \tau_{t}.$$
(14)

Now consider the future relative yield spreads. From Equations 9, 10, 11, and 12, it can be shown that

$$r_{1t} - r_{1t}^{\mathfrak{r}} [1 - E_t(\tau_{t+1})] = \operatorname{Cov}_t [u'(c_{t+1}), r_{1t}^{\mathfrak{r}} (1 - \tau_{t+1})] [E_t u'(c_{t+1})]^{-1},$$
(15)

and

$$r_{2t} - r_{2t}^{\mathfrak{r}} \left[1 - E_t(\tau_{t+2}) \right] = \operatorname{Cov}_t \left[u'(c_{t+2}), r_{2t}^{\mathfrak{r}} (1 - \tau_{t+2}) \right] \left[E_t u'(c_{t+2}) \right]^{-1}.$$
(16)

Equations 15 and 16 can be rearranged, respectively, as

$$(r_{1t}^{t} - r_{1t}) / r_{1t}^{t} = E_{t}(\tau_{t+1}) - \Delta_{1} / r_{1t}^{t}, \qquad (17)$$

where $\Delta_1 = \operatorname{Cov}_t[u'(c_{t+1}), r_{1t}^{\mathsf{r}}(1 - \tau_{t+1})][E_t u'(c_{t+1})]^{-1}$, and

$$\left(r_{2l}^{T}-r_{2l}\right)/r_{2l}^{T}=E_{l}(\tau_{l+2})-\Delta_{2}/r_{2l}^{T},$$
(18)

where $\Delta_2 = \operatorname{Cov}_{l}[u'(c_{l+2}), \tau_{2l}^{\tau}(1-\tau_{l+2})][E_{l}u'(c_{l+2})]^{-1}$.

Equations 17 and 18 indicate that there are two major factors affecting the future relative yield spreads. The first factor is the expected value of the future tax rate. It is noted that τ_{i+1} and τ_{i+2} are unknown at time *t*. Given the setup of the model, $E_t(\tau_{i+i}) = \tau_i$ for i = 1, 2. This simply says that when the information about the future tax rate is not in the information set, using the (indifference) tax rate to form expectations is the optimal choice.

The second factor is the scaled covariance between the future marginal utility derived from the future consumption and the future tax rate,

$$\Delta_i / r_{ii}^x = \operatorname{Cov}_i [u'(c_{i+i}), r_{ii}^x (1 - \tau_{i+i})] \{ [E_i u'(c_{i+i})] r_{ii}^x \}^{-1},$$
(19)

where i = 1, 2. Because $E_i u'(c_{i+i})$ for i = 1, 2 is a positive term based on the monotonic assumption of u(c), the sign of Δ_i is dependent on the sign of the covariance. It is known that a higher (lower) future tax rate τ_{i+i} which is dependent upon a higher (lower) future income, leads to a lower (higher) future after-tax yield, $r_{ii}^{\tau} (1 - \tau_{i+i})$, which may affect the future consumption in the following two ways.⁴

CASE 1

Case 1 (the income effect) is the case where a lower (higher) future after-tax yield makes the future consumption lower (higher) and hence the marginal utility derived from it becomes higher (lower). In this case, the terms Δ_i (i = 1, 2) are less than zero. $E_t(\tau_{\mu,i}) = \tau_t$ for i = 1, 2, and Equations 14, 17, and 18 imply that the future relative yield spreads are higher than the current relative yield spread:

$$(r_{1+1}^{\tau} - r_{1+1}) / r_{1+1}^{\tau} < (r_{1}^{\tau} - r_{1}) / r_{1\nu}^{\tau}$$

$$(20)$$

and

$$(r_{1 \leftarrow 1}^{\mathfrak{r}} - r_{1 \leftarrow 1}) / r_{1 \leftarrow 1}^{\mathfrak{r}} < (r_{2\iota}^{\mathfrak{r}} - r_{2\iota}) / r_{2\iota}^{\mathfrak{r}}.$$

$$\tag{21}$$

Based on the above inequalities, it is clear that the term structure would be downward-sloping if

$$\Delta_1/r_{1t}^{t} < \Delta_2/r_{2t}^{t} < 0.$$

As term-to-maturity increases, relative yield spreads would approach the (indifference) tax rate from above. Hence, we have $\Delta_2 / \Delta_1 > r_{2t}^x / r_{1t}^x$. Since r_{2t}^x usually exceeds r_{1t}^x , i.e., the yield curve of taxable bonds is normally upward-sloping, $\Delta_2/\Delta_1 > r_{2t}^x/r_{1t}^x > 1$ reflects the fact that the income effect is greater in a more distant future because of greater future uncertainty.

CASE 2

Case 2 (the substitution effect) is the case where a lower (higher) future after-tax yield leads more (less) investment in other possibilities (such as bonds with shorter terms-to-maturity or that are tax-exempt). Such portfolio rebalancing makes the future consumption higher (lower) and the marginal utility derived form it becomes lower (higher). In this case, the terms Δ_i (i = 1, 2) are greater than zero. $E_i(\tau_{i+i}) = \tau_i$ for i = 1, 2, and Equations 14, 17, and 18 imply that the future relative yield spreads are lower than the current relative yield spread:

$$(r_{1 \leftarrow 1}^{t} - r_{1 \leftarrow 1}) / r_{1 \leftarrow 1}^{t} > (r_{1 t}^{t} - r_{1 t}) / r_{1 t}^{t}$$

$$(22)$$

and

$$(r_{1 \leftarrow 1}^{\mathfrak{x}} - r_{1 \leftarrow 1}) / r_{1 \leftarrow 1}^{\mathfrak{x}} > (r_{2t}^{\mathfrak{x}} - r_{2t}) / r_{2t}^{\mathfrak{x}}.$$

$$\tag{23}$$

As can be derived from the above inequalities, the term structure would be downward-sloping if

$$\Delta_2/r_{2t}^{\rm T} > \Delta_1/r_{1t}^{\rm T} > 0.$$

As term-to-maturity increases, relative yield spreads would *fall way* from the (indifference) tax rate. Hence, we have $\Delta_2/\Delta_1 > r_{2t}^r/r_{1t}^r$. Since r_{2t}^r usually exceeds r_{1t}^r , $\Delta_2/\Delta_1 > r_{2t}^r/r_{1t}^r > 1$ reflects the fact that the substitution effect is greater in a more distant future because of greater future uncertainty.

In summary, the theoretical conjecture suggests that the persistent pattern of the downward-sloping term structure could potentially result from income and substitution effects. When the income effect dominates, relative yield spreads would approach the (indifferent) tax rate from above. When the substitution effect dominates, the spreads would fall from the tax rate. The income uncertainty and hence tax rate uncertainty in a more distant future lead to greater income and substitution effects. Since relative yield spreads tend to fall away from the (indifference) tax rate as

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term-to-maturity increases, it can be inferred that this may reflect mainly the current and future substitution effects, and the investors' efforts to rebalance their portfolio in the face of future income uncertainty. The substitution effect in the model may also be reflected in the various degrees of market segmentation observed in real life.⁵

III. CONCLUSION

This paper extends the analysis of the impact of income uncertainty on relative yield spreads between taxable and tax-exempt bonds based on the investor's dynamic portfolio choice model. The analysis indicates that the slope of the term structure of relative yield spreads could be related to the investors' decisions and corresponding income and substitution effects. The persistent pattern of the term structure may be largely due to the substitution effect since relative yield spreads tend to fall away from the (indifference) tax rate as term-to-maturity increases.

As is noted, the downward-sloping term structure started to flatten from the late 1980s and this continued gradually into the 1990s (see Table 1). Within the proposed dynamic portfolio choice model, this tendency may be due to the decreasing importance of substitution effects. It is also possible that other factors such as institutional changes may have come into play. This relatively recent phenomenon obviously represents a new challenge for future research.

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NOTES

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1. The representative agent model is also limited in that it is not a suitable framework for studying heterogeneous behaviors among taxable and tax-exempt bond investors.

2. As can be seen later, the selection of one- and two-period coupon bonds is made for simplicity and the analysis in this model could be fairly general.

3. Duffie (1988) provides a general proof of the existence of a unique solution of a more general Markovian model.

4. The author would like to thank one referee for very stimulating comments on the two cases.

5. See Fama (1977), and Mitchell and McDade (1992), among others.

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