Regime Dependent Sensitivity of Country Exchange Traded Funds to Common Risk Factors

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Abstract

If common factors jointly affect country stock markets, it is an indication of the global stock market integration. Common factors may affect some markets more/less than other markets, an indication of the degree of the global stock market integration/segmentation. In this paper, we study the intergration of global stock markets based on the returns on exchange traded funds (ETFs) for the U.S., Canada, U.K., Germany, France, Italy, Australia and Japan. The relationship between country ETF returns and common risk factors may be time-varying across countries, and that favors a regime switching (RS) factor model for the dynamics of the country ETF returns. A RS factor model for the relationship between country ETF returns and common risk factors is fitted to daily data for period from May 31, 2000 to March 31, 2014. We use the data to test a hierarchy of hypotheses on country ETF returns: (1) common factor exposure across some country ETFs and all regimes, and (3) common factor exposure across some country ETFs and all regimes. The RS factor model for ETF returns fits the data well and the common factors have variable effects across countries and over regimes.

JEL Codes: C13, C32, G12

Keywords: country exchange traded funds, common risk factors, regime switching models

1 Introduction

International asset pricing has long been an interest in the finance literature (see, for example, Cho et al. (1986), Berkaert and Harvey (1995), Errunza and Losq (1985), Foerster and Karolyi (1999), Heston et al. (1995), Jorion and Schwartz (1986), Korajczyk and Viallet (1989), Longin and Solnic (1995), De Santis and Gerard (1997), and Solnik (1974a, 1974b, 1983)). A focus of these papers is the degree to which international stock markets are segmented or integrated. Barriers to investing in foreign markets have been reduced and the result is an increase in international financial cross-holdings. In the aggregate, international asset trade has grown more rapidly than international trade in goods and services (Lane and Milesi-Ferretti, 2003). This is not to suggest there is a single world market. There is a substantial global component to local stock market performance, but the variation in structural and geopolitical conditions contributes to variation. Furthermore, there is a country by country difference in the level of activity in the various market sectors, and differences in country portfolios could reflect differences in composition. The questions are then what common/country specific factors affect these markets, and what models can best describe the links between the markets and the factors.

The trend to increasingly integrated international stock markets and co-movements between the rates of return of foreign and domestic assets might limit the benefits of diversification with foreign investment. The picture is not simple. Foerster and Karolyi (1999) suggest that the market segmentation hypothesis can be confirmed based returns on non-U.S. firms cross-listed on U.S. exchanges as American Depositary Receipts (ADR's). Cross-listings achieve cumulative abnormal returns initially, although there is a subsequent reversion, and share price changes are robust to changing market risk exposures. There is little evidence of a permanent effect on returns for firms' listings in markets that are more liquid, provide better legal protection, or have a larger shareholder base (Sarkissian and Schill, 2009).

There are distinct forces for integration and segmentation. International stock markets share common risk factors which drive co-movement and integration (Heston et al., 1995). At the same time, local market conditions (e.g. liquidity, legal restrictions, and market participation) foster segmentation. The degree of integration/segmentation depends on those forces, which vary across geographically separate markets and over time.

Most cost effective and readily available foreign investment vehicles, relative to individual foreign stocks or country specific mutual funds, are country specific exchange traded funds (ETFs) that mimic country stock market indices. Evidence indicates that ETF's closely track their underlying indices, with tracking errors generally below 1 percent (Agapova, 2011; Andreu et al., 2013). They have become very common vehicles that investors use to diversify their portfolios. With respect to the return on investment in country ETFs, there are several issues of concern: (i) the common risk factors (such as the global stock market portfolio return) affecting the returns of country ETFs, which drive integration; (ii) the variation in the sector composition of country ETFs, which supports segmentation (such as economies biased towards resources or manufacturing); and (iii) investors' evaluation of and decision on purchasing/disposing of assets in different states of nature, in which large gains and large losses occur (MacLean, Zhao and Ziemba, 2013). Therefore, a suitable asset pricing model for country ETF returns should take these issues into consideration.

The factors which affect asset returns have been widely studied. Levy and Lieberman (2013) use high frequency data to identify the reaction of country specific ETFs to the S & P 500 index. In addition to the stock market index, other factors include the book-tomarket ratio (Fama and French, 1992, 1993, 1996, and 1998; Hou et al., 2011), debt-equity ratio (Bhandari, 1988; Hou et al., 2011), earning-to-price ratio (Basu, 1977; Fama and French, 1992), size (Banz, 1981; Fama and French, 1992, 1993, 1996, and 1998), momentum (Jegadeesh and Titman, 1993; Carhart, 1997; Chordia and Shivakumar, 2002; Hou et al., 2009), stock market volatility (Black, 1976; French et al., 1987; Glosten et al., 1993; Ghysels et al., 2005; Ang et al., 2006; Koulakiotis et al., 2006), yield spread (Chen et al., 1986; Campbell, 1987; Fama and French, 1989), credit spread (Chen et al., 1986; Keim and Stambaugh, 1986), commodity price (Johnson and Soenen, 2009), oil price (Chen et al., 1986; Jones and Kaul, 1996; Sadorsky, 1999; Basher and Sadorsky, 2006), exchange rate (Solnik, 1974a, 1974b; Roll, 1992; Dumas and Solnik, 1995; Ferson and Harvey, 1999), and the Baltic Dry index for global shipping cost (Bakshi et al., 2011).

Economists have used different models to relate asset returns to relevant factors. Basu

(1977) and Fama and French (1992) use static models. Login and Solnik (1995) and De Santis and Gerard (1997) suggest that the suitable model could be a conditional variance model such as GARCH. Bekaert and Harvey (1995) propose the two regime model for studying common and country factors in asset pricing. But when multiple country assets are considered, is it possible that more regimes exist? Bos and Newbold (1984) find that betas may not be constant over time, suggesting that a pricing model with time-varying features may shed more light on asset pricing. To incorporate time-varying features into asset pricing, Ferson and Harvey (1991) propose a two-step conditional asset pricing model. Following their work, Jagannathan and Wang (1996) and Ferson and Harvey (1998) find that the conditional model is more convincing than the traditional model with constant betas. Fridman (1994), Schaller and Norden (1997), Assoe (1998), and Liu et al. (2011) use regime switching (RS) models, allowing market regimes and regime-dependent betas. One major distinction among these models is the variability of betas. While both the conditional asset pricing model and RS model allow time-varying betas, the latter combines the time-varying and state-dependent features, which may shed more light on asset pricing. On one hand, Lewellen and Nagel (2006) find that the conditional asset pricing model performs as poorly as the traditional static model. On the other hand, Fridman (1994), Schaller and Norden (1997), Assoe (1998) and Liu et al. (2011) find that equity returns exhibit strong regime switching behaviours over time.

In this study, eight iShares country ETFs are studied with reference to a set of common risk factors in a RS model. This paper differs from the literature in a number of ways. First, this paper studies the country ETF pricing with a unique set of common factors. Second, various unobservable market regimes are inferred directly from the country ETF and common factor data . Third, although some studies incorporate the regime switching feature in pricing asset returns, few estimate these models with a joint distribution on all asset returns.

Using a RS factor model for the vector of returns on these country ETFs provides a setting for investigating specific hypotheses about these country markets. The extent of integration/segmentation will be manifested in the nature of the relationships to common factors. For example, if country markets are weakly related to the aggregate world market, then markets are distinct. Alternatively, if the same factors are significant in country markets then they are a force for integration.

The rest of the paper is organized as follows. In Section 2, we discuss the country ETF returns and the common risk factors potentially effecting the country ETF returns. In Section 3, we explain the RS model and the hypotheses concerning these country ETF returns with model parameters being restricted. Section 4 presents the empirical findings. Section 5 concludes.

2 Country ETF Returns and Risk Factors

In this section, we focus on the key variables of interest, country ETF returns, for the study of international financial markets, and potential common risk factors which could affect the pricing of international assets. Evidence shows that ETF's track the underlying indices. As shown in Table 1, the composition of the US ETF and that of the S&P500 are very similar.

(Please insert Table 1 about here.)

2.1 Country ETF Returns

In order to diversify investment globally, investors can either invest in foreign assets directly or buy the shares of international funds. The former involves higher costs that offsets the benefit of diversification while the latter achieves the diversification globally at lower costs. These high costs are from two sources. First, the costs of collecting adequate information for specific assets are high for individual investors. Second, transaction costs of investing directly in foreign assets are also high. Among international funds are passively managed country ETFs, which mimic market portfolio indices composed of assets from various market sectors, or country mutual funds. There is some evidence that country mutual funds cost more and are less efficient.¹ If the composition varies by country, then the country ETFs provide

¹It is generally true that country mutual funds cost more. Harper et al. (2006) show that the close-end country funds also cost more.

investors with low cost and various international diversification opportunities. In addition, they are readily tradable in the major stock markets such as the U.S. stock market.

The objective of this paper is to study country ETFs as investable international assets. Since there are a number of country ETFs in the U.S. market, we choose the following criteria to select our research sample. (1) All the selected country ETFs should be managed by the same company to maintain portfolio consistency. (2) The trading history must be long enough so to have large samples for estimation. (3) The ETFs must be liquid in the sense that their shares are actively traded. (4) The ETFs must be priced in a benchmark currency, the U.S. dollars, in the study.²

Among all country ETFs in the market, only iShares international index funds satisfy the above criteria. We choose eight developed market ETFs which account for a substantial portion of the total global market capitalization. These eight countries account for 76.52% of the MSCI all country world investable market index (ACWI IMI) in value. This index is designed to capture up to 99% of the developed and emerging investable market universe. See file "ACWI IMI factsheet" on the website of MSCI Inc. The selected ETFs are for the United States (US), Canada (CA), United Kingdom (UK), Germany (GER), France (FRA), Italy (ITA), Australia (AUS), and Japan (JAP). The daily dividend-adjusted closing prices of all eight country ETFs from May 30, 2000 to March 31, 2014 are retrieved from quandl.com. The difference in the natural logarithm of prices is taken to get the daily returns for these ETFs — $ln(P_{t+1}) - ln(P_t) = ln[P_{t+1}/P_t] = ln[1 + R_t] \approx R_t$. Hence, we have the daily returns from May 31, 2000 to March 31, 2014. The returns over fixed periods of the eight funds are our key variables of interest, which are expressed as as a vector of 8 ETF returns at period t:

$$R_t = [R_{1t}, \dots, R_{8t}]'.$$
(1)

²This ensures that the U.S. dollar price of an non-U.S. asset reflects both the underlying value in the domestic currency and the impact of the exchange rate.

2.2 Risk Factors

The common risk factors considered for their relationship to country ETF returns are listed in Table 2. Discussion of the factors follows, and expectation for the sensitivity of country ETF rates of return to factor levels and/or changes in factor levels over a fixed period is provided.

(Please insert Table 2 about here.)

As implied by the CAPM, asset returns are systematically related to overall market returns. Thus, the return of the total market should be priced into asset returns. In the paper, the MSCI All Country World Investable Market Index (WOD) is used as the proxy of the world stock market. This index covers over 9,000 securities across large, mid and small cap size segments and across style and sector segments in 45 developed and emerging stock markets.³ The daily closing prices during the period from May 30, 2000 to March 31, 2014 are retrieved from Datastream. We use the log-difference to calculate returns from May 31, 2000 to March 31, 2014.

Market volatility is another prominent factor although empirical findings on the effect on asset returns are somewhat mixed. French et al. (1987) and Ghysels et al. (2005) find a positive premium of market volatility on the U.S. value weighted portfolio. But Glosten et al. (1993) find a negative premium of market volatility on the U.S. stock market return. Ang et al. (2006) also find that market volatility is negatively associated with the U.S. stock mean return. Koulakiotis et al. (2006) find no significant relation between the returns of stock market indices and market volatility for seven OECD countries. Liu et al. (2011) show that, within a RS model for the U.S. sectoral ETFs, the signs of sensitivities of sectoral ETF returns to market volatility vary across sectors and market regimes and this may explain the inconsistency in reported results. Currently there is a consensus that the higher (lower) the volatility measured by VIX, the more (less) pervasive fear that exits in the market. Following Liu et al. (2011), we use the Chicago Board Options Exchange Volatility Index (VIX) as

 $^{^3\}mathrm{See}$ file "ACWI IMI factsheet" on the website of MSCI Inc.

the proxy for market volatility. The daily index data from May 31, 2000 to March 31, 2014 are retrieved from Datastream. We use the log-difference to calculate percentage changes.

Foreign exchange rates are an important factor for international asset pricing. Roll (1992) compares stock price indices across countries and finds that exchange rates play a significant role in explaining the returns of stock market indices represented by a common currency. Dumas and Solnik (1995) find evidence for an exchange rate risk premium. Ferson and Harvey (1999) find that currency risk factors are important in pricing developed market returns. Exchange rates not only have impact on the fundamentals of international assets across countries but also affect the asset values traded in the U.S., where investors can still trade the underlying assets included in these country ETFs before the U.S. market opens and after it closes. Hence, exchange rates may explain in part the returns of these country ETFs. We take the trade-weighted U.S. dollar index (DXY) as the proxy for exchange rates in relation to the U.S. dollar. The daily index data from May 31, 2000 to March 31, 2014 are retrieved from Datastream. We use the log-difference to calculate percentage changes. This index measures the value of the U.S. dollar against a basket of foreign currencies. An increase of the index indicates that the U.S. dollar appreciates against other currencies.

Material-related factors are also studied in international asset pricing. Jones and Kaul (1996) and Sadorsky (1999) find a negative impact of the oil price on real stock returns. Johnson and Soenen (2009) find that changes in the S&P Goldman Sachs Commodity Index (GSCI) can explain a small part of variation in stock market returns in Argentina, Brazil, Chile, Colombia and Peru. Bakshi et al. (2011) find that the growth rate of the Baltic Dry index (BDI) — an index for shipping costs— is positively associated with stock market returns in G7 countries. The existing literature has also documented significant risk premiums on material-related factors such as the commodity index, oil price and Baltic Dry index (Chen et al., 1986; Johnson and Soenen, 2009; Bakshi et al., 2011). These three factors (oil price, commodity prices and the BDI) are closely related to the changes in oil price. As an indicator of ocean transportation costs, the Baltic dry index changes simultaneously in response to the changes of oil price. Oil products account for more than 60% of the S&P

GSCI in value.⁴ Since the commodity price index not only takes into account oil products but also encompasses other raw materials such as metals, softs, etc, it could be a better candidate for a risk factor. Therefore, we use the percentage change of the S&P Goldman Sachs Commodity Index (COM) as the commodity risk factor. It is a world production weighted index based on the quantity of production of each commodity. The daily index data from May 31, 2000 to March 31, 2014 are retrieved from Datastream. We use the log-difference to calculate percentage changes.

As found by Frankel (1993), Chang et al. (1995), and Russell (1998) and Gutierrez at al. (2009), U.S. exchange traded foreign assets exhibit a significant exposure to U.S. market factors and behave like U.S. securities. We expect that the U.S. exchange listed country ETFs may also be exposed to some U.S. common risk factors. Fama and French (1992) find that three factors (size, book-to-market and market) can explain most of the anomalies in returns except for the momentum effect. (Also see Fama and French (1992, 1993, 1996, and 1998). Part of their findings contradicts Banz (1981) that "there is little difference in return between average sized and large firms." Despite that contradiction, the Fama-French three factors prevail in subsequent studies and are applied to new investment vehicles such as sectoral ETFs. For instance, Liu et al. (2011) and Ma et al. (2011) apply the Fama-French factors to pricing the returns on sectoral ETFs in the U.S. market and find that these factors exhibit strong explanatory power. "Small minus big" (SMB) for size and "High minus low" (HML) for value are average return differences over a fixed period. The SMB and HML data are retrieved from Kenneth R. French data library from May 31, 2000 to March 31, 2014.⁵

Yield spread (YS) and credit spread (CS) are two economic indicators. Chen et al. (1986) find that stock returns are negatively related to YS, whereas some other studies (e.g. Campbell, 1987; Fama and French, 1989) document positive risk premiums on YS. Keim and Stambaugh (1986) find a positive risk premium on CS, while Fama and French (1993) find that CS factor is not significant. Liu et al. (2011) find that the sensitivities of sector ETF returns to YS and CS factors vary across market regimes. Considering the variation of sector composition of country ETFs, we include yield spread (YS) and credit spread (CS) in

⁴Data source: S&P indices website. Link: http://www.standardandpoors.com/indices/sp-gsci/en/us.

 $^{^{5}}$ Data source: http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/index.html.

our asset pricing model. YS is defined as the difference between the 30-year U.S. Treasury bond and the 3-month U.S. Treasury bill and CS is defined as the difference between return of Moody's Baa bond and return of Moody's Aaa bond. The data from May 31, 2000 to March 31, 2014 are retrieved from Datastream.

In addition, a number of previous studies have documented a significant momentum effect. Some studies find that the momentum effect is due to stock price overreaction (e.g. Jegadeesh and Titman, 1993; Hou et al., 2009), while some other studies find no such phenomenon (e.g. Carhart, 1997). Chordia and Shivakumar (2002) find that the momentum effect can be explained by macroeconomic factors of the business cycle. More specifically, they find that the momentum strategy only generates positive returns during the expansionary period while it generates insignificant negative returns during recession. Their findings suggest that different market regimes may potentially capture effects that have been captured by momentum.

3 Model Specification and Hypotheses

It is anticipated that the dynamics of the returns on country ETFs are affected by the dynamics of common risk factors. This section discusses the specification of a RS model relating the returns on the selected eight country ETF to the proposed risk factors. Relationship hypotheses are defined from model parameters and the methods for estimation of parameters and testing hypotheses are presented.

3.1 Regime Switching Factor Model

In the basic statistical analysis of the eight country ETF returns, it is noted that the distributions for these returns have third and fourth moments that are far from those of the normal distribution. These returns are either positively or negatively skewed and have fat tails. Mixtures of normal distributions can explain these phenomena, and a Markov RS model where the transition probabilities provide the mixing weights is a modeling option. Furthermore, it is noted that that investors tend to overreact to the information in the stock market and hence this behavioral bias could cause asset prices to deviate from equilibrium levels depending on certain market conditions (e.g. Jegadeesh and Titman, 1993; Hou et al., 2009; MacLean, et. al., 2013). The RS model provides the flexibility and structure for plausible hidden market regimes and links country ETF returns to common risk factors jointly while accommodating return skewness and fat tails as the result of mixtures of conditional distributions.

To explain the RS model, we assume that variables are defined over discrete time intervals or periods, t = 1, ..., T. Let $R_t = [R_{1t}, R_{2t}, ..., R_{Nt}]'$ be the vector of N ETF returns in period t and $Z_t = [Z_{1t}, Z_{2t}, ..., Z_{Kt}]'$ be the vector of K common factors in period t affecting R_t . Let S_t be the regime (or state) variable in period t which takes one of M discrete values in period t such at $s_t = j$ with $j \in \{1, 2, ..., M\}$. In this paper, we permit all returns to be modeled jointly with the shared regimes and shared changes in regimes over time. Thus the data generating process for all ETF returns are jointly estimated in a multivariate framework. We define this RS factor model for the vector of the returns on N country ETFs, $R_t = [R_{1t}, R_{2t}, \ldots, R_{Nt}]'$, conditional on regime s_t in period t as:

$$R_t = Z_t \beta_{s_t} + P_{s_t} U_t, \tag{2}$$

where Z_t is a matrix which has the same set of K common factors in each row in period t, with Z_{0t} being a vector of unity:

 S_t is the market regime variable in period t taking on values $s_t = j, j \in \{1, 2, ..., M\}$. The vector of beta coefficients in period t with regime j are for N ETFs and K common risk

factors:

$$\beta_{s_t=j} = [\beta_{10j}, \beta_{11j}, \dots, \beta_{1Kj}, \beta_{20j}, \beta_{21j}, \dots, \beta_{2Kj}, \dots, \beta_{N0j}, \beta_{N1j}, \dots, \beta_{NKj}]'.$$
(4)

 $U_t = [u_{1t}, u_{2t}, \ldots, u_{Nt}]'$ is a vector of error terms and $U_t \sim N(0, I)$. Here, we assume that there is no autocorrelation for u_{it} for all i.⁶ However, u_{it} and u_{jt} can be correlated for all $i, j = 1, 2, \ldots, N$. The variance-covariance matrix conditional on the market regime s_t is be expressed as

$$\Sigma_{s_t} = P_{s_t} P'_{s_t} = \begin{bmatrix} \sigma_{11,s_t} & \sigma_{12,s_t} & \dots & \sigma_{1N,s_t} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{N1,s_t} & \sigma_{N2,s_t} & \dots & \sigma_{NN,s_t} \end{bmatrix}.$$
(5)

Consider the residual errors $X_t = P_{s_t}U_t = R_t - Z_t\beta_{s_t}$, which are a function of observable ETF returns and common factors. Let $X^{(t)} = [X_1, X_2, \ldots, X_t]$ and $S^{(t)} = [S_1, S_2, \ldots, S_t]$ be the series of residuals and regimes from period 1 to period t. We assume that X_t and S_t satisfies the following Markov properties. First, the probability distribution of the current state, S_t , only depends on the state in the previous period, S_{t-1} , and the transition probability in period t - 1:

$$\Pr(S_t|S^{(t-1)}) = \Pr(S_t|S_{t-1}).$$
(6)

Second, the probability distribution of X_t only depends on S_t :

$$\Pr(X_t | X^{(t-1)}, S^{(t)}) = \Pr(X_t | S_t).$$
(7)

Third, the regime dynamics are stationary. This can be characterized by the transition probability matrix Γ :

$$\Gamma = \begin{bmatrix} \gamma_{11} & \dots & \gamma_{1M} \\ \vdots & \ddots & \vdots \\ \gamma_{M1} & \dots & \gamma_{MM} \end{bmatrix},$$
(8)

⁶When fitting the model for the data, the common practice is to use an information criterion to identify the number of the regimes and the number of the factors suitable to the RS model to avoid any specification errors. The model found through this practice often gets errors close to be independent over time.

where $\gamma_{ji} = \Pr(s_t = i | s_{t-1} = j)$ is the transition probability that market changes from regime $s_{t-1} = j$ in period t-1 to regime $s_t = i$ in period t for all t's.⁷

3.2 Hypotheses

The RS model contains a set of relationship parameters $B = \{\beta_{ikj}\}$, for N ETFs (i = 1, ..., N), K factors (k = 1, ..., K), and M regimes (j = 1, ...M), which is of interest for research purposes. The initial questions concern the dimensions of the model: Are the country ETF returns determined by different data generating processes? Are there multiple regimes embedded in the data generating process? Are all common risk factors relevant in the data generating process?

If the country funds are defined by investment weights in market sectors,

$$ETF_i$$
's weights = $[w_{i1}, ..., w_{iL}],$ (9)

where w_{il} is the fraction in sector l in country i's fund. Stacking these vectors of weights for all L countries yields

$$W = \begin{bmatrix} w_{11} & \dots & w_{1L} \\ \vdots & \ddots & \vdots \\ w_{N1} & \dots & w_{NL} \end{bmatrix}.$$
 (10)

The variation by country in sector weights would be important in interpreting the effect of factors on the returns for country funds. It is possible that by sector country ETF weights are all equal (to the global index weights) and in that case the factor WOD in Z_1 would fully explain the structure for returns on country funds.

There is evidence for a regime structure for returns on sector funds (Liu et al., 2011). Variation in sector weights for country funds could alter the regime structure implied by the sector funds. Assuming that market forces (law of one price) dictate that sector fund returns are equal across countries, we let sector fund returns in period t be $V_t = [v_{1t}, ..., v_{Lt}]'$. Then the total return in period t is $R_t \approx WV_t$, and the regime structure for V_t could be

⁷These transition probabilities satisfy the equation $\sum_{i=1}^{M} \gamma_{ji} = 1$ for all i, j = 1, 2, ..., M.

transformed in the RS model for country funds, since the factor coefficients in the country fund model would be weighted combinations of the coefficients in the RS model for sector funds. Preliminary analysis indicates there is a multiple regime model for sector funds, and similar sector weights for many developed countries supports a multiple regime model for country ETF returns. The appropriate number of regimes will be determined based on data using an information criterion.

The common risk factors under consideration have support in referenced studies on asset pricing dynamics. There are two parts to the changes in asset returns, the dynamics of the within regime country ETF returns and the switching between regimes by these country ETF returns. The RS model defines the within regime ETF returns as a linear function of changes in the common risk factors. The regime switching probabilities are not direct functions of these factors. (There is some evidence that dramatic shifts in price dynamics (regime shifts) are linked to thresholds/levels on common risk factors.) In assessing the common risk factors, all the period by period changes, regardless of regimes (in single regime), are considered. The interrelationship among changes in these factors, and the relationship between changes in these factors and country ETF returns are analyzed. Factors that are not related to country ETF returns when all periods, regardless of regimes (in single regime), are considered are likely not important for within regime changes nor for regime switching.

Assuming that the chosen model contains all relevant common risk factors and a suitable number of regimes as shown later in the paper, we then can impose restrictions on sensitivity parameters of the country ETF returns to changes in the factors within a regime or across regimes in the restricted model while the full model without any restriction is named the unrestricted model. The restricted model under the null hypotheses can be then tested with reference to the unrestricted model under the alternative hypotheses. Since the unrestricted and restricted models are nested, we can use the standard likelihood-ratio tests because the standard asymptotic theory of likelihood-ratio tests is valid for hidden Markov models (Giudici, Ryden, and Vandekerkhove, 2000), as long as non-standard situations such as testing parameters on the boundary (see Dannemann and Holzmann, 2008) are not of concern.

Our approach is to consider separately the effect of each factor on the ETF returns. A

sequence of null and alternative hypotheses is proposed, with the next null hypothesis is conditional by the alternative of the previous null hypothesis: (1) no factor effect on all ETF returns in all regimes; (2) no factor effect on specific ETF returns in all regimes; (3) no factor effect on a specific ETF return in a specific regime. It is important to note that the alternative to (3) is that there is an effect of the factor on the specific ETF return in the specific regime, so that (4) no factor effect on all ETF returns in the specific regime is tested.

3.2.1 Common factor exposure across all country ETFs and all regimes

In the mulple regime model, some common factors have no effect across all country ETF returns and all regimes as illustrated in Figure 1.

(Please insert Figure 1 about here.)

If the matrix of coefficients for factor k across all ETFs and regimes is $\beta_{\cdot k \cdot}$, then the competing hypotheses are

$$H_k^0: \ \beta_{\cdot k \cdot} = 0 \ vs \ H_k^a: \ \beta_{\cdot k \cdot} \neq 0.$$

Examples of the null hypotheses of this kind are as follows: (1) The world stock market factor has no effect on country ETF returns for all countries and in all regimes. (2) The market volatility has no effect on country ETF returns for all countries and in all regimes. In this setting, the restricted model under H_k^0 has all factors included except Z_k while the unrestricted model under H_k^a has all factors included.

3.2.2 Common factor exposure across some country ETFs and all regimes

If the matrix of coefficients $\beta_{\cdot k}$ is not the null matrix, it is still possible that factor k has no effect on one or more of the ETFs. So focusing on an ETF as illustrated in Figure 2 gives the ETF specific hypotheses.

(Please insert Figure 2 about here.)

Let β_{ik} be the vector of coefficients across regimes for ETF *i* and factor *k*. The competing hypotheses are

$$H_{ik}^0: \ \beta_{ik} = 0 \ vs \ H_{ik}^a: \ \beta_{ik} \neq 0.$$

Examples of the null hypotheses of this kind are as follows: (1) The commodity price has no effect on the resource rich country's (e.g., Australia and Canada) ETF returns in all regimes. (2) The U.S. dollar has no effect on the non-U.S. country ETF returns in all regimes. (3) The U.S. dollar has no effect on the U.S. country ETF return in all regimes.

3.2.3 Common factor exposure across some country ETFs and some regimes

Drilling down further, it is possible that factor k has no effect for some ETF returns in some regime. That is, the coefficient for factor k in the sub-model for country i's ETF return in regime j may not be statistically significant.

The hypotheses corresponding to Figure 3 are

$$H^0_{ikj}: \beta_{ikj} = 0 \text{ vs } H^a_{ikj}: \beta_{ikj} \neq 0.$$

For example, the commodity index may have no effect on the ETF return in Italy in bear markets.

(Please insert Figure 3 about here.)

3.3 Model Fitting

Consider the observations (r_t, z_t) in period t, t = 1, ..., T for the country ETF returns R_t and common factors Z_t . Define $x_t = r_t - z_t \beta_{s_t}$, which depends on the unknown parameters β_{S_t} and Σ_{S_t} . Let $x^{(t)} = [x_1, ..., x_t]$ and $s^{(t)} = [s, ..., s_t]$. Assume that the number of distinct states/regimes is M: $s_t = j, j \in \{1, 2, ..., M\}$. The EM algorithm is used to estimate model parameters. The mathematical specification of the RS model is based on Zucchini and MacDonald (2009).

The complete data likelihood function (CDLL) is defined as follows:

$$L_{T} = \Pr(X^{(T)} = x^{(T)}) = \sum_{s_{1}, s_{2}, \dots, s_{T}=1}^{M} \Pr(X^{(T)} = x^{(T)}, S^{(T)} = s^{(T)})$$
(11)
$$= \sum_{s_{1}, s_{2}, \dots, s_{T}=1}^{M} \left[\delta_{s_{1}} \prod_{t=2}^{T} \gamma_{s_{t-1}, s_{t}} \prod_{t=1}^{T} p_{s_{t}}(x_{t}) \right]$$
$$= \delta \mathbf{P}(x_{1}) \Gamma \mathbf{P}(x_{2}) \dots \Gamma \mathbf{P}(x_{T}) \mathbf{1}',$$

where δ is a row vector of the initial probability distribution of all states {Pr($s_1 = 1$), Pr($s_1 = 2$),..., Pr($s_1 = M$)}; γ_{s_{t-1},s_t} represents the transition probability from state s_{t-1} to state s_t while Γ is the transition probability matrix defined earlier; $p_{s_t}(x_t) = \Pr(X_t = x_t|s_t)$; $\mathbf{1} = [1, 1, ..., 1]_{1 \times M}$, and $\mathbf{P}(x_t) = \text{diag}\{p_1(x_t), ..., p_M(x_t)\}$.

We use the CDLL with the observations, x_1, x_2, \ldots, x_T , and the missing data (latent random variable), s_1, s_2, \ldots, s_T , to estimate all parameters in the model. The CDLL becomes

$$\log[\Pr(X^{(T)} = x^{(T)}, S^{(T)} = s^{(T)})]$$
(12)
= $\log\left(\Pr(s_1)\prod_{t=2}^{T}\Pr(s_t|s_{t-1})\prod_{t=1}^{T}\Pr(x_t|s_t)\right)$
= $\log\left(\delta_{s_1}\prod_{t=2}^{T}\gamma_{s_{t-1},s_t}\prod_{t=1}^{T}p_{s_t}(x_t)\right)$
= $\log\delta_{s_1} + \sum_{t=2}^{T}\log\gamma_{s_{t-1},s_t} + \sum_{t=1}^{T}\log p_{s_t}(x_t).$

In this setup, there are three sets of parameters:

- 1. The initial probability of state $j: \{\delta_{s_1}\};$
- 2. Transition probabilities: $\{\gamma_{s_{t-1},s_t}\};$
- 3. Sensitivity parameters (or intercept and slope parameters) $\{\beta_{s_t=j}\}$ and variance covariance matrix of random errors of the model $\{\Sigma_{s_t=j}\}$.

Define two zero-one random variables. (1) One represents the sequence of states:

$$\{s_1, s_2, \dots, s_t\}\tag{13}$$

such that $u_j(t) = 1$ if $s_t = j$ and $u_j(t) = 0$ otherwise. (2) The other represents the transition between states in the next period: $v_{ji}(t) = 1$ if $s_{t-1} = j$ and $s_t = i, t = 1, 2, ..., T$ and $i, j = 1, 2, ..., M; v_{ji}(t) = 0$ otherwise.

Then, the CDLL can be written as

$$\log L_T = \sum_{j=1}^M u_j(1) \log \delta_j + \sum_{j=1}^M \sum_{i=1}^M \left(\sum_{t=2}^T v_{ji}(t) \right) \log \gamma_{ji} + \sum_{j=1}^M \sum_{t=1}^T u_j(t) \log p_j(x_t).$$
(14)

The values of $v_{ji}(t)$ and $u_j(t)$ are unknown and must be estimated in addition to the linear model parameters.

E step:

- 1. Assign initial values for all parameters $\hat{\Theta} = [\{\hat{\delta}_j\}, \{\hat{\gamma}_{ji}\}, \{\hat{\Sigma}_{s_t=j}\}, \{\hat{\beta}_{s_t=j}\}$ for all $i, j = 1, 2, \ldots, M$].
- 2. Use the initial values of the parameters to compute the conditional expectations of $v_{ji}(t)$ and $u_j(t)$:

$$\hat{u}_j(t) = E[u_j(t)|x^{(T)}, \hat{\Theta}] = \Pr(s_t = j|x^{(T)}) = \alpha_t(j)\rho_t(j)/L_T$$
(15)

$$\hat{v}_{ji}(t) = E[v_{ji}(t)|x^{(T)}, \hat{\Theta}] = \Pr(s_{t-1} = j, s_t = i|x^{(T)}) = \alpha_{t-1}(j)\gamma_{ji}p_i(x_t)\rho_t(i)/L_T.$$
 (16)

In equations (15) and (16), $\alpha_t(j) = Pr[X^{(t)} = x^{(t)}, s_t = j]$ and $\rho_t(j) = Pr[X_{t+1}^T = r_{t+1}]$

 $x_{t+1}^T | s_t = j]$, where $X_{t+1}^T = (X_{t+1}, ..., X_T)$. So $\hat{u}_j(t)$ and $\hat{v}_{ji}(t)$ are the *a posteriori* probabilities, based on Bayes law, for the state and transition functions respectively.

M step:

1. Replace $v_{ji}(t)$ and $u_j(t)$ by $\hat{v}_{ji}(t)$ and $\hat{u}_j(t)$ in the CDLL.

2. Maximize the CDLL w.r.t those three sets of parameters Θ .

We can split this process into three separate maximizations:

First, the term $\sum_{j=1}^{M} \hat{u}_j(1) \log \delta_j$ depends only on $\{\delta_j\}$. The solution is

$$\delta_j = \hat{u}_j(1) / \sum_{j=1}^M \hat{u}_j(1) = \hat{u}_j(1).$$
(17)

Second, the term $\sum_{j=1}^{M} \sum_{i=1}^{M} \left(\sum_{t=2}^{T} \hat{v}_{ji}(t) \right) \log \gamma_{ji}$ depends only on γ_{ji} . The solution is

$$\gamma_{ji} = f_{ji} / \sum_{i=1}^{M} f_{ji}, \qquad (18)$$

where $f_{ji} = \sum_{t=2}^{T} \hat{v}_{ji}(t)$.

Third, the term $\sum_{j=1}^{M} \sum_{t=1}^{T} \hat{u}_j(t) \log p_j(R_t - Z_t \beta_{s_t=j})$ depends only on $\{\Sigma_{s_t=j}\}$ and $\{\beta_{s_t=j}\}$. It can be written as follows:

$$\sum_{j=1}^{M} \sum_{t=1}^{T} [\hat{u}_{j}(t) \log p_{j}(\mathbf{P}_{s_{t}}^{-1}R_{t} - \mathbf{P}_{s_{t}}^{-1}Z_{t}\beta_{s_{t}=j})]$$
(19)
$$= \sum_{j=1}^{M} \sum_{t=1}^{T} \hat{u}_{j}(t) \log \left(\frac{1}{(2\pi)^{n/2} |\Sigma_{s_{t}=j}|^{1/2}} e^{-\frac{1}{2}[R_{t} - Z_{t}\beta_{s_{t}=j}]'\Sigma_{s_{t}=j}^{-1}[R_{t} - Z_{t}\beta_{s_{t}=j}]}\right)$$
$$= \sum_{j=1}^{M} \sum_{t=1}^{T} \hat{u}_{j}(t) \left(-\frac{n}{2} \log 2\pi - \frac{n}{2} \log |\Sigma_{s_{t}=j}| - \frac{1}{2}(R_{t} - Z_{t}\beta_{s_{t}=j})'\Sigma_{s_{t}=j}^{-1}(R_{t} - Z_{t}\beta_{s_{t}=j})\right)$$

Please note that the update equations for the mean vector and covariance matrix in (19) are a weighted form of the standard maximum likelihood functions for linear models. Maximizing

the concentrated log-likelihood function given by equation (19) is equivalent to minimizing the generalized least squares across regimes. The exact solution is given by the weighted least squares estimates for $\{\Sigma_{s_t}\}$ and $\{\beta_{s_t}\}$. In addition, robust standard errors for $\{\beta_{s_t}\}$ will be important for statistical inference.

The expectation and maximization (EM) steps are repeated for various numbers of iterations until convergence is obtained.⁸ The process requires initial parameter estimates. In practice, given that we know the number of hidden regimes and which common factors affect the country ETF returns, we use randomly selected initial parameter estimates for $\{\hat{\delta}_j\}$ and $\{\hat{\gamma}_{ji}\}$ from the multinomial distribution and for $\{\hat{\Sigma}_{s_t}\}$ and $\{\hat{\beta}_{s_t}\}$ from the Gaussian distribution. The EM algorithm selects the estimation results that produce the highest value of the CDLL.

Clearly, in the factor model described by equation (2), we need to decide the number of hidden regimes that can be optimally identified for the eight country ETF returns and decide which common factors are viewed as valid factors affecting the country ETF returns. When we increase the number of regimes and/or more factors for the model, the number of parameters to be estimated increases exponentially. To balance the goodness of fit and parsimony, the Bayesian information criterion (BIC) is used to choose the suitable model with an optimal selection of K common factors and M regimes. The BIC is given by

$$BIC = -2\log(L_T) + p\log(n), \tag{20}$$

where p is the number of model parameters in $\hat{\Theta}$ and n = TN is the number of observations. In addition to the BIC, we could also use the likelihood test to evaluate the model that is best supported by the data.

4 Empirical Results

The study of country ETFs and the impact of common risk factors on their returns is based on the daily returns on 8 iShares international index funds for the United States (US), Canada

⁸The tolerancy level is set to $1e^{-8}$ for the relative change in the log-likelihood function.

(CA), United Kingdom (UK), Germany (GER), France (FRA), Italy (ITA), Australia (AUS), and Japan (JAP) and the daily values of 8 common risk factors between March 31, 2000 and March 31, 2014. The relationship between the country ETF returns and common risk factors is expected to be regime-dependent.

4.1 ETF Returns Distribution

The summary statistics for the country ETF returns are reported in Table 3. The summary statistics include the number of observations (No. of obs), minimum, maximum, mean, median, variance, standard deviation (Stdev), skewness and kurtosis for the ETFs for the U.S., Canada, U.K., Germany, France, Italy, Australia, and Japan. As shown in Table 3, during the period from May 31, 2000 to March 31, 2014, Australia and Canada had, respectively, the first and second highest mean returns while Japan and Italy had, respectively, the first and second lowest mean returns. The standard deviations of these ETF returns are fairly comparable. The Japan ETF return has a positive skewness while all other country ETF returns are negatively skewed. The country ETF returns all demonstrate fat tails with kurtosis ranging from 5.34 for France to 10.18 for the U.K. Clearly, these returns are not normal. It is possible that these returns are better modeled as a mixture of normal distributions. The RS model, therefore, may be a better choice.

(Please insert Table 3 about here.)

The correlation matrix for rates of return in Table 4 shows how these country ETF returns are pair-wise correlated. Among all pairs of country ETFs, the Canada and Japan ETF returns have the lowest correlation (0.57) while the France and Germany ETF returns have the highest correlation (0.91). The Japan ETF return has lower correlations with all other country ETFs ranging from 0.57 to 0.69. Other country ETF returns have higher pair-wise correlations among themselves ranging from 0.69 to 0.91. This indicates that there is co-movement in international asset returns, which may be the result of the effects from common market factors.

(Please insert Table 4 about here.)

Since the returns on different sector portfolios behave differently in response to changes of common risk factors (see Liu et al., 2011), studying the compositions of country ETFs would help explain the joint distribution of these eight country ETF returns. The sector weights in these country ETFs and the World Index are listed in Table 1, where the variation in composition is evident. First, the Germany, Japan, U.S. and France ETFs invest significantly in the consumer discretionary sector. Since this sector mainly provides non-essential goods and services, it tends to perform well when the market performs relatively well. Thus, the performance of these four country ETFs is more likely to show strong links to the world stock market. Second, the U.S., Canada, U.K., France, Italy and Australia ETFs invest significantly in the financial sector. The performance of these ETFs may be subject to changes of interest rate, financial market sentiment and so on. Third, the Canada, U.K. and Italy ETFs significantly invest in energy. This indicates that changes of energy prices may have a positive impact on returns of these ETFs. Fouth, Australia, Canada, Germany and U.K. ETFs invest heavily in materials. It implies that an increase in prices of raw materials may lead to higher returns of these ETFs.

4.2 Statistics for Common Risk Factors

In the RS model, common risk factors serve as the conditioning variables which are expected to affect country ETF returns. The basic statistics and correlations are relevant to interpretation of the model. The basic statistics for the common risk factors are summarized in Table 5.

(Please insert Table 5 about here.)

The focus is on the sensitivity of the country ETF returns to changes in the levels of the common risk factors, so standardized daily changes (percentage change) in these factors are considered. The exception is the spreads (CS, YS), which are measured by the basis points. Over the period of study, changes in the common risk factors are considerable. The maximums and minimums are very different. The interrelationship between the common risk factors is given in Table 6. The correlations indicate the market common risk factors (WOD, VIX, DXY, and COM) are interrelated. The Fama French factors (SMB and HML) are largely independent of other factors. The similar observation can be made for the credit and yield spreads (CS and YS). An analysis of the correlation structure shows that WOD and VIX have the highest negative correlation.

(Please insert Table 6 about here.)

4.3 Model Selection

The RS model relates the country ETF return rates to a set of common risk factors across regimes. The analysis is iterative, so that the model with the best set of common risk factors and the optimal number of regimes is identified. The pair-wise unconditional correlations between the common risk factors and country ETF returns are reported in Table 7. The correlations between WOD and VIX and country ETF returns are high and quite consistent across all countries. Considering the composition of the country ETF portfolios, it is expected that WOD and VIX play significant roles. The correlations between DXY and COM and ETF returns are not as high. The correlations between SMB and HML and ETF returns are much weaker while the correlations between CS and YS and ETF returns are weakest.

(Please insert Table 7 about here.)

To further investigate the effect of factors, a single regime model for all eight country ETF returns with all eight common risk factors (SMB, HML, WOD, VIX, DYX, COM, CS, and YS) is estimated. The likelihood ratio test statistics indicate that we cannot reject the null hypothesis that the coefficients associated with CS and YS factors are jointly zeros across all eight countries (see Tables 8 and 9). The fact that the spreads are levels whereas the other factors measure changes could account for the non-significance. The coefficients of the other six common risk factors are statistically significant.

(Please insert Tables 8 and 9 about here.)

We use the BIC to decide on the number of regimes and the number of common risk factors of the RS model jointly. In the preliminary analysis of the single regime factor models for country ETF returns, it is identified that some factors, such as CS and YS, are statistically insignificant for all country ETF returns while some other factors, such as COM, SMB, and HML, are statistically insignificant for some country ETF returns. We use the six factor RS model as the baseline model that contains WOD, VIX, DXY, COM, SMB, and HML. Other possible RS models are specified by removing, respectively, COM, SMB, HML, COM + SMB, COM + HML, HML + SMB, or COM + HML + SMB from the baseline model, or by adding, respectively, CS, YS, or CS + YS to the baseline model.

Table 10 shows the BIC values of these candidate RS models with various numbers of common risk factors and various numbers of regimes. Among all these BIC values, the lowest BIC value (-193620.10) is associated with the six factor RS model with six regimes. This confirms the results of the single regime model that excludes the CS and YS factors. It is also noteworthy that he six factor model is preferred to the eight factor model that includes CS and YS regardless of the number of regimes.

(Please insert Table 10 about here.)

4.4 Fitted Model: Market Regimes and Transition Probabilities

The best fitted model has six regimes and six common risk factors, which are SMB, HML, WOD, VIX, DYX, and COM. The regimes are a stochastic dynamic process for states of nature, with transitions between regimes on successive days. We will give broad interpretations to the market regimes by considering the transition probabilities in Table 11 and the mean risk factors and mean ETF returns across regimes as reported in Tables 12 and 13.

For ordering regimes a contiguity principle is assumed. That is, regimes are close/distant based on the major common risk factors (e.g., WOD and VIX), and transition to close regimes is more probable than transition to distant regimes. Of course there will be exceptional times when a market shock will move factors and returns quickly and dramatically rather than incrementally.

The transition matrix in Table 11 indicates the regimes are structured (illustrated in bold font). There is cycling with regime 1 and regime 2, and also with regime 3 and regime 4. For regime 5 and regime 6, the chance of remaining in the regime is high.

(Please insert Tables 11 about here.)

The six regimes of the RS model are shared by all six country ETF returns conditional on the six common risk factors. This is consistent with the fact that the U.S. stock market is the largest in terms of market capitalization and that all six country ETFs are actually traded in the U.S. stock market. Therefore, we consider the major common risk factors for intuition in giving meanings to regimes. The major common risk factors are WOD and VIX. Based on positive/negative returns on the world stock market index and decreasing/increasing volatility (as given by Table 12) the six regimes are broadly classified into bull (regime 1), weakly bull (regime 2), transition 1 (regime 3), transition 2 (regime 4), weakly bear (regime 5), and bear (regime 6) as indicated in Tables 11 and 12.

(Please insert Table 12 about here.)

The mean returns on the country ETFs across regimes are given in Table 13 . There is some, but not absolute, similarity in the mean returns on country ETFs in in each of these regimes. We can look to the global market portfolio index (WOD) for indications of market integration. This global index is composed largely of assets from the countries represented in the country ETFs. The U.S. assets alone comprise almost 52% of the global index. Considering the similarity in the sector composition between the global index and many country ETFs (see Table 1), the returns on the global index (WOD) and some country ETFs (US, CAN, and AUS) should be similar (see Table 13). However, there remains some variation with some other ETFs. It remains to be seen if integration/segregation among all country ETFs to the common risk factors across, and by, regime. For this purpose, the characteristics of the RS model for all eight ETFs and its sub-models for various individual ETFs will be further explored in the next subsection.

(Please insert Table 13 about here.)

4.5 Hypothesis Testing: Impact of Factors on Returns across Regimes for Country ETFs

The global financial market transitions through various regimes over time, and the common risk factors and country ETF returns vary accordingly. The dynamics within a regime are described by the linear factor model. The effect of common risk factors could be different by regime, a reflection of investment decisions and prices reacting to market conditions. Of particular interest are differences in the effects of the common risk factors on different country ETFs and whether such differences depend on the extant regime. The hypotheses presented in Subsection 3.2 are effect/no effect statements which need to be tested. These tests provide useful information about the hypotheses and give clear guidance on the interpretation of the role that common risk factors play across different country ETFs and across different regimes.

4.5.1 Common factor exposure across all country ETFs and all regimes

As we have indicated previously in the paper, the credit and yield spreads (CS and YS) are not statistically significant in the single regime model and cannot be selected for multiple regime models based on the BIC. Now we can evaluate the statistical significance of the remaining factors (WOD, VIX, DYX, COM, SML and HML) in the RS model by testing the following null hypothesis against the alternative hypothesis, $H_k^0: \ \beta_{\cdot k \cdot} = 0 \ vs \ H_k^a: \ \beta_{\cdot k \cdot} \neq 0.$ We use the likelihood ratio test to test the null hypothesis under which $\beta_{k} = 0$ restrictions are imposed on sensitivity coefficients associated with one common risk factor for all eight country ETF returns across all six regimes. The test statistic follows the χ^2 distribution with 48 degrees of freedom. The testing results are provided in Table 14. The global index return (WOD) has much greater impact than other factors. The test statistics is 7176.00 with the p-value that approaches 0. The market volatility (VIX) and the US dollar index (DXY) are also highly statistically significant, with the test statistics of 1231.45 and 1386.19 respectively. The test statistics for the commondity index (COM) is 531.68 with the pvalue that approaches 0. The size and value factors, SML and HML, are also statistically significant with the test statistics of 311.12 and 385.29, respectively. Therefore, we can conclude that all six factors are statistically significant in the full model for all eight country ETF returns and that the country ETF returns are determined by all six commom risk factors across all regimes. This indicates that there is a substantial market integration among the stock markets in the U.S., Canada, U.K., Germany, France, Italy, Australia, and Japan. Although, as noted previously, the stock market in Japan is weakly correlated with the markets in the rest of the eight countries (see Table 4), when the eight country ETF returns are jointly examined, a systematic dynamics of asset pricing pattern emerge amount these eight country ETFs. This reinforces the evidence of market integration.

4.5.2 Common factor exposure across some country ETFs and all regimes

Although a factor is significant in the full model in which the sub-models for all country ETF returns are included, it is possible that a factor's contribution to a specific country ETF is not statistically significant in a corresponding sub-model within the full model. To test the hypothesis of this kind, we use the following null hypothesis and alternative hypotheses for each factor k and country i: H_{ik}^0 : $\beta_{ik.} = 0$ vs H_{ik}^a : $\beta_{ik.} \neq 0$. That is, under the null hypothesis, a particular factor is removed from a sub-model for a particular ETF return across all regimes. In this context, we use the likelihood ratio test statistic. The test results are provided in Table 15. The likelihood ratio test statistic has 6 degrees of freedom. The test statistics and their *p*-values are listed. We use the 5% significance level ($\alpha = 0.05$) to judge if the null hypothesis should be rejected.

- As indicated by Table 15, the major common risk factors WOD and VIX are statistically significant for all country ETFs, although the strength for Japan is considerably weaker than for other countries. Once again, this is an indication of the global market integration among the U.S., Canada, U.K., Germany, France, Italy, Australia, and Japan.
- 2. Table 15 shows that the dollar index DXY is statistically significant for all country ETFs except the Australia ETF (at the 5% significant level). Perhaps, this is due to the fact that as a major raw material exporting country the dollar index plays a major role in asset pricing—that is, the weaker the dollar, the greater the Australia ETF return. This is an indication that the global market is somewhat segmented.

- 3. The commodity index COM is statistically significant for all countries as shown Table 15. The likelihood ratio test statistic is variable, with this factor consistently strong in the U.S., Canada and U.K. but not so strong in other countries. Even though one would expect the commodity index COM may play' different roles in different countries, we do not see a strong evidence for this conjecture.
- 4. Table 15 indicates that the size factor SMB is statistically significant for all country ETFs. The test statistics are high for the U.S., Canada, U.K., Germany and Italy but are lower for France, Australia, and Japan.
- 5. As shown in Table 15, the value factor HML is statistically significant for all country ETFs except the Japan ETF. Similarly the test statistics is variable across all country ETFs.

While the above discussion is based on removing one factor for a country across six regimes at a time, in the following we test some specific hypotheses by removing a factor among some country ETFs but not among others. Now we exlain these tests below.

1. We formally test the null hypothesis that the commodity price has no effect on the resource rich country's (e.g., Australia and Canada) ETF returns in all regimes. The likelihood-ratio test statistics is 388.67 with the *p*-value that approaches 0. Therefore, we can reject the null hypothesis. This test result confirms our conjecture that the performance of some ETFs that tracks resources rich economies (e.g. Canada and Australia) benefits from rising commodity prices. As shown in Table 1, the materials and energy sectors account rfor 37.39%, 26.20%, and 21.54% of Canada, U.K. and Australia ETFs in value, respectively. Given an increase in commodity prices, the profit gained from materials and energy sectors outweighs the loss from the other sectors. As a result of the net gain resulting from higher commodity prices, the value of underlying assets goes up and thus leads to higher returns of these resources-rich country ETFs. As for the other non-resources-rich country ETFs, it works in the opposite direction. This indicates that the global market is segmented along the lines of those who are more sensitive or less sensitive to the commodity index.

2. We formally test the null hypothesis that the U.S. dollar has no effect on the non-U.S. country ETF returns but have an effect on the U.S. country ETF return in all regimes. The likelihood-ratio test statistics is 1553.05 with the *p*-value that approaches 0. Therefore, we can reject the null hypothesis. This indicates that the global market is segmented along the lines of those who are more sensitive or less sensitive to the dollar index. The non-U.S. country ETF returns are affected by the U.S. dollar index.

4.5.3 Common factor exposure across some country ETFs and some regimes

Taking the restricted model a bit further, we also consider the impact of dropping a factor from the RS model for a country ETF in one regime. The tested hypotheses for each country i, each factor k and each regime j are H_{ikj}^0 : $\beta_{ikj} = 0$ vs H_{ikj}^a : $\beta_{ikj} \neq 0$. Figure 4 plots the estimated coefficients for each of the factors for each country across 6 regimes. Since each of the factors is measured as a percent change, the estimated coefficients (sensitivities) are comparable.

The results of tests on the individual coefficients are presented in Table 16, which is a summary of Tables 17–24. The 1% significant level is used to indicate statistical significance (S) or statistical non-significance (N). In other words, "S" represents that the sensitivity coefficient is statistically significant while "N" represents that the sensitivity coefficient is statistically insignificant. Each cell consists of six letters that represents the statistically significance across 6 regimes. For example, the risk factor WOD is statistically significant for all country ETFs across all 6 regimes. Hence we see SSSSSS. But the commodity price risk factor COM is statistically insignificant for the France (FRA) and Italy (ITA) country ETFs across all 6 regimes. Hence we see NNNNNN.

The restricted model from the hypothesis H_{ikj}^0 : $\beta_{ikj} = 0$ only drops one factor in one regime for one ETF, so the t test statistic is often small.

From the results in Figure 4 and Table 16 we observe the following:

1. WOD: The coefficient is statistically significant in all countries and regimes. The global stock market return WOD has a positive impact on the returns of all eight country ETFs. This is consistent with the existing studies. We further observe that

the magnitude of this impact varies across different regimes. In addition, the sensitivity coefficient estimates for most country ETFs are a bit higher in the bear market than in the bull market. This implies that these country ETFs become more sensitive to the global stock market when the times are bad. This evidence is an indication of the global market integration.

- 2. VIX: The VIX risk factor is derived from the implied volatility in the S&P 500 index option and is a measure of fear in the U.S. stock market. Our estimation results of the RS model show that almost all country ETF returns are negatively associated with market volatility. But the strength of association varies across six regimes. The pattern across regimes is similar for most countries. The coefficient is statistically significant for some regime in all countries and usually in most regimes. In Regime 5 the VIX factor is not significant in most countries. Also in Japan it is not significant in most regimes. That is, the Japan ETF appears more segregated from the other country ETFs.
- 3. DXY: The U.S. dollar index (the DXY factor) reflects the terms of trade between the U.S. and its trading partners. The change of this index would have different implications to different countries. An increase (decrease) in the DXY factor represents appreciation (depreciation) of the U.S. dollar against other currencies. Our estimation results of the RS model shows a positive relationship between the DXY factor and U.S. ETF return but a negative relationship between the DXY factor and remaining seven country ETF returns. A positive relationship between the DXY factor and U.S. ETF return suggests that appreciation of the U.S. dollar relative to other currencies is often driven by the same forces (good economic fundamentals in the U.S. relative to other countries) that increase U.S. asset prices, generating higher returns for the U.S. ETF. A negative relationship between the DXY and each of other seven non-U.S. ETF returns suggests that appreciation of the U.S. dollar relative to other currencies is often driven by the same forces (good economic fundamentals in the U.S. relative to other countries) that increase U.S. asset prices, generating higher returns for the U.S. ETF. A negative relationship between the DXY and each of other seven non-U.S. ETF returns suggests that appreciation of the U.S. dollar relative to other currencies is often driven by the same forces (good economic fundamentals in the U.S. relative to other countries) that decrease non-U.S. asset prices, generating lower returns for non-U.S.

ETFs. With the exception of Japan The DXY factor is statistically significant in some regime and usually in most regimes. In regime 6 (bear) the factor is not significant. This evidence indicates that the U.S. dollar index is one factor that segments the global market.

- 4. COM: The commodity price affects different countries differently as some are resource exporting countries while others are resource importing countries. For Canada, Australia and UK, their country ETF returns respond positively to increases of commodity prices, while the returns of the other country ETFs are neutral or negative. The incremental effect of COM is in general negligible, but it is important to note that the global index WOD partly accounts for commodity prices. Correspondingly, the COM coefficient is not statistically significant for most regimes in all countries.
- 5. SMB: The coefficients for the SMB factor are quite erratic and the incremental effect is often not significant. The exception is the US where the size effect is positive and significant.
- 6. HML: The effect of the value factor is almost always positive, but the incremental effect is usually not statistically significant. The correlation with other factors diminishes the significance of HML.

5 Conclusion

In this paper, we use a RS factor model and eight country ETFs to analyze the integration and segmentation of the global stock market. The ETFs are for the countries U.S., U.K., Canada, Germany, France, Italy, Australia, and Japan. The factors are the global stock market, market volatility, U.S. dollar index, commodity prices, size, value, credit spread, and yield spread. Based on the BIC, we select the suitable number of regimes and relevant common risk factors. We identify six market regimes: bull (regime 1), weakly bull (regime 2), transition 1 (regime 3), transition 2 (regime 4), weakly bear (regime 5), and bear (regime 6) and six relevant common risk factors: the global stock market, market volatility, U.S. dollar index, commodity prices, size, and value. The RS model jointly estimated for the six ETF returns is flexible enough to allow the identification of market integration/segmentation. This is an improvement over the model without regimes.

From various statistical tests on the RS model, we note that: (1) six common risk factors (the global stock market, market volatility, U.S. dollar index, commodity index, size, and value) are statistically significant; (2) the six common risk factors are significant for each of the country ETFs, although there is some variation in the level of significance; (3) the coefficients (sensitivities) relating ETF returns to common risk factors also vary by regime. In some regimes for some ETFs the coefficients are not statistically significant.

With respect to the coefficients associated with the common risk factors, it is observed that the all country global stock market return (WOD) has a positive impact on our eight country ETF returns, although the magnitude varies across regimes and countries. The market volatility (VIX) is negatively correlated with returns of all eight country ETFs across all regimes. However, the country ETF returns tend to be more sensitive to market volatility in the bull market. These suggest that the global stock market is highly integrated. The U.S. dollar index (DXY factor) is priced into the returns of these eight country ETFs. The DXY factor contributes a positive premium on the U.S. ETF return and negative premiums on the returns of all non-U.S. country ETFs across regimes. The returns of the U.K., Canada and US ETFs are positively correlated with changes in the commodity price index (COM) factor) in most market regimes while the returns of all other country ETFs have a neutral to negative relation to changes in the COM factor in most market regimes. These different sensitivities to the U.S. dollar index and commodity prices indicate that the global stock market is also segmented along the lines of the U.S. dollars and commodity prices. We find that the size and value factors can partially explain the returns of these eight county ETFs, but the impact of these two factors on the country ETFs varies in direction and magnitude across different market regimes.

The implications of the results from the RS model for market integration/segmentation are consistent with expectations. The strongest global factors (WOD, VIX) have a major effect on all country portfolio's and are a powerful force for market integration. There is some variation in the scale of the sensitivity coefficient but the pattern is similar for countries and regimes. At the same time, the direction and size of the impact from COM, SMB and HML factors depend on the country and in some cases the regime. These differentiating factors are significant, although not as important as the integrating factors.

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Figure 3: Specific Factor and Regime Effect





Figure 4: Factor Coefficients by Regime for Country ETF Returns

Notes: Each panel tracks the sensitivity coefficients of eight country ETFs (the U.S., Canada, U.K., Germany, France, Italy, Australia, and Japan) to a risk factor (WOD, VIX, COM, DXY, SML, and HML) across 6 regimes (1 for "bull", 2 for "weakly bull", 3 for "transition 1", 4 for "transition 2", 5 for "weakly bear", and 6 for "bear").

	$_{\rm US}$					ETFs				
Sector	S&P500	U.S.	Canada	U.K.	Germany	France	Italy	Australia	Japan	World
Consumer Discretionary	12.12%	12.62%	5.23%	8.17%	21.92%	13.34%	10.74%	1.98%	20.78%	13.00%
Consumer Staples	9.7%	9.00%	3.00%	16.29%	3.95%	7.09%	-	9.07%	6.56%	8.98%
Energy	10.31%	10.16%	25.78%	16.83%	-	11.19%	20.30%	3.56%	1.24%	7.20%
Financials	16.36%	16.00%	36.72%	21.28%	16.44%	17.95%	35.21%	50.86%	19.47%	21.70%
Health Care	13.41%	12.20%	3.81%	8.97%	4.14%	10.60%	-	6.68%	6.44%	11.58%
Industrials	10.69%	10.41%	7.02%	7.18%	14.31%	18.43%	13.65%	5.42%	19.64%	11.43%
Information Technology	18.56%	18.81%	1.19%	1.10%	7.36%	3.02%	-	0.50%	10.93%	13.94
Materials	3.48%	3.49%	11.61%	9.37%	14.24%	6.06%	-	17.98%	5.64%	5.66%
Telecommunication Service	2.48%	2.67%	2.65%	5.42%	3.90%	5.00%	4.19%	1.95%	5.65%	3.26%
Utilities	3.07%	3.05%	1.29%	4.23%	4.38%	4.28%	15.88%	0.94%	2.50%	3.16%
Others/Undefined	-	1.60%	1.69%	1.16%	9.36%	3.04%	0.02%	1.05%	1.15%	-

Table 1: Comparison of Sector Weights across S&P 500 and Country ETFs

Notes: Each number in the table represents the weight of a sector held by the corresponding ETF. The symbol "-" indicates that the ETF does not hold any securities from the corresponding sector. The data are collected from the factsheet of each ETF and Standard&Poor website as of March 31, 2014.

Factor	Description
Z_1 :WOD	The rate of return on the MSCI World Market Index
Z_2 :VIX	The percentage change of the CBOE Volatility Index
Z_3 :DXY	The percentage change of the U.S. Dollar Index
$Z_4: \operatorname{COM}$	The percent change in the Goldman Sacks Commodity Index
Z_5 :SMB	The average return difference between F & F small and big portfolios
Z_6 :HML	The average return difference between F & F value and growth portfolios
Z_7 :YS	The yield difference between 20-year U.S. T bond and 3-month U.S. T bill
$Z_8: \mathrm{CS}$	The yield difference between Moody's Baa And Aaa bonds

Table 2: Definitions of Commmon Risk Factors

Table 3: Basic Statistics for Country ETF Returns

					v			
	US	CA	UK	GER	\mathbf{FRA}	ITA	AUS	JAP
No. of obs	3479	3479	3479	3479	3479	3479	3479	3479
Minimum	-0.0951	-0.116660	-0.127770	-0.119870	-0.115970	-0.111813	-0.132100	-0.109019
Maximum	0.09935	0.116590	0.157450	0.180220	0.123210	0.142665	0.188160	0.146788
Mean	0.00017	0.000280	0.000160	0.000170	0.000140	0.000081	0.000450	-0.000017
Median	0.00075	0.000950	0.000570	0.001020	0.000580	0.000754	0.000980	0.000000
Variance	0.00017	0.000240	0.000250	0.000330	0.000310	0.000349	0.000330	0.000232
Stdev	0.01296	0.015520	0.015760	0.018090	0.017690	0.018685	0.018260	0.015232
Skewness	-0.24045	-0.465440	-0.183550	-0.037620	-0.159440	-0.184734	-0.165990	0.098368
Kurtosis	6.93089	5.687370	10.177690	7.186800	5.341310	5.377446	9.445470	6.566957

Notes: These dividend adjusted country ETF returns are from May 31, 2000 to March 31, 2014.

 Table 4: Correlations among Country ETF Returns

	US	CA	UK	GER	FRA	ITA	AUS	JAP
US	1.000000							
CA	0.730000	1.000000						
UK	0.800000	0.710000	1.000000					
GER	0.810000	0.690000	0.830000	1.000000				
\mathbf{FRA}	0.790000	0.710000	0.840000	0.910000	1.000000			
ITA	0.720000	0.670000	0.780000	0.830000	0.880000	1.000000		
AUS	0.710000	0.700000	0.720000	0.710000	0.730000	0.690000	1.000000	
JAP	0.690000	0.570000	0.660000	0.660000	0.650000	0.590000	0.620000	1.000000

Notes: The data for country ETF returns are from May 31, 2000 to March 31, 2014.

Table 5: Basic Statistics for Common Risk Factors

	Table	U. Dasi	, Duanau		ummon i	ubr rac	1015	
	WOD	VIX	DXY	COM	SMB	HML	\mathbf{CS}	YS
No. of obs	3479	3479	3479	3479	3479	3479	3479	3479
Minimum	-0.03181	-0.15226	-0.01327	-0.03972	-3.79000	-4.91000	0.59000	-0.52850
Maximum	0.03951	0.21541	0.01094	0.03133	4.30000	3.95000	3.47000	4.69290
Mean	0.00003	-0.00007	-0.00005	0.00013	0.01743	0.02367	1.11025	2.69064
Median	0.00027	-0.00210	-0.00004	0.00035	0.03000	0.02000	0.96000	3.12870
Variance	0.00002	0.00076	0.00001	0.00004	0.33045	0.39758	0.22763	2.22423
Stdev	0.00473	0.02760	0.00230	0.00666	0.57485	0.63054	0.47710	1.49139
Skewness	-0.31492	0.64719	-0.05329	-0.26053	-0.11607	-0.02520	2.82454	-0.75756
$\operatorname{Kurtosis}$	7.25057	4.29188	1.38050	2.55192	3.79546	6.74409	8.99975	-0.77289

Notes: The data for percentage changes/returns of the common risk factors are from May 31, 2000 to March 31, 2014.

Table 6: Correlations among Common Risk Factors

				0				
	WOD	VIX	DXY	COM	SMB	HML	\mathbf{CS}	YS
WOD	1.00000	-0.67878	-0.29233	0.34382	0.03876	0.08190	-0.00111	0.01468
VIX	-0.67878	1.00000	0.11486	-0.19644	-0.12783	-0.04023	-0.01936	-0.01441
DXY	-0.29233	0.11486	1.00000	-0.28199	-0.04337	-0.12706	-0.00854	-0.00870
COM	0.34382	-0.19644	-0.28199	1.00000	0.01351	0.14114	-0.01811	0.00259
SMB	0.03876	-0.12783	-0.04337	0.01351	1.00000	-0.10508	0.00898	0.02239
HML	0.08190	-0.04023	-0.12706	0.14114	-0.10508	1.00000	-0.03571	-0.02728
\mathbf{CS}	-0.00111	-0.01936	-0.00854	-0.01811	0.00898	-0.03571	1.00000	0.26170
YS	0.01468	-0.01441	-0.00870	0.00259	0.02239	-0.02728	0.26170	1.00000

Notes: The data for percentage changes/returns of the common factors are from May 31, 2000 to March 31, 2014.

Table 7: Correlations among Country ETF Returns and Common Risk Factors

				v				
	US	CA	UK	GER	FRA	ITA	AUS	JAP
WOD	0.88648	0.76522	0.83492	0.85239	0.85171	0.77816	0.74606	0.71837
VIX	-0.75087	-0.56701	-0.63494	-0.63897	-0.64639	-0.59931	-0.58345	-0.54812
DXY	-0.12693	-0.34558	-0.32354	-0.38159	-0.40543	-0.44512	-0.36123	-0.21347
COM	0.24620	0.44135	0.31025	0.27848	0.29951	0.30208	0.34930	0.19693
SMB	0.11255	0.09904	0.01662	0.06580	0.05741	0.08149	0.06656	0.05689
HML	0.06424	0.12761	0.14971	0.10720	0.15380	0.19792	0.20011	0.04948
\mathbf{CS}	-0.00329	0.00347	-0.00235	-0.00053	0.00075	-0.00183	0.00070	0.00603
\mathbf{YS}	0.01557	0.00471	0.00736	0.00402	0.00361	0.00079	0.00288	0.01296

Notes: The data for country ETF returns and percentage changes/returns of the common factors are from May 31, 2000 to March 31, 2014.

	US	UK	CA	GER	FRA	ITA	AUS	JAP
(Intercept)	0.0003	0.0002	0.0001	0.0004	0.0002	0.0002	0.0003	-0.0002
	(0.0004)	(0.0006)	(0.0006)	(0.0006)	(0.0005)	(0.0006)	(0.0008)	(0.0006)
WOD	2.0908^{***}	2.3373^{***}	1.9580^{***}	2.7901^{***}	2.6181^{***}	2.3379^{***}	2.1483^{***}	2.1151^{***}
	(0.0503)	(0.0812)	(0.0810)	(0.0902)	(0.0648)	(0.0769)	(0.1099)	(0.0892)
VIX	-0.1152^{***}	-0.0846^{***}	-0.0595^{***}	-0.0862^{***}	-0.0979^{***}	-0.1110^{***}	-0.1109^{***}	-0.0600^{***}
	(0.0059)	(0.0083)	(0.0090)	(0.0107)	(0.0089)	(0.0111)	(0.0125)	(0.0105)
DXY	0.6744^{***}	-0.6352^{***}	-0.6773^{***}	-1.2804^{***}	-1.4108^{***}	-1.9704^{***}	-1.1427^{***}	-0.1634
	(0.0569)	(0.0894)	(0.0865)	(0.0928)	(0.0897)	(0.1026)	(0.1073)	(0.1031)
COM	-0.0662^{***}	0.0102	0.4211^{***}	-0.1314^{***}	-0.0877^{***}	-0.0534	0.1837^{***}	-0.1304^{***}
	(0.0212)	(0.0342)	(0.0324)	(0.0348)	(0.0315)	(0.0363)	(0.0426)	(0.0337)
SMB	0.0013^{***}	-0.0007^{**}	0.0016^{***}	0.0006^{*}	0.0003	0.0013^{***}	0.0009^{**}	0.0005
	(0.0003)	(0.0005)	(0.0004)	(0.0006)	(0.0004)	(0.0004)	(0.0006)	(0.0005)
HML	0.0004^{**}	0.0018^{***}	0.0011^{***}	0.0009***	0.0020***	0.0035^{***}	0.0036^{***}	-0.0000
	(0.0003)	(0.0004)	(0.0004)	(0.0003)	(0.0004)	(0.0005)	(0.0005)	(0.0004)
CS	-0.0002	-0.0001	0.0003	-0.0000	0.0000	-0.0000	0.0002	0.0001
	(0.0004)	(0.0006)	(0.0005)	(0.0006)	(0.0005)	(0.0005)	(0.0007)	(0.0006)
YS	0.0000	-0.0000	-0.0001	-0.0001	-0.0001	-0.0001	-0.0001	0.0000
	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)
\mathbb{R}^2	0.8447	0.7209	0.6420	0.7585	0.7707	0.6873	0.6146	0.5261
Adj. R ²	0.8444	0.7202	0.6412	0.7579	0.7702	0.6866	0.6137	0.5250
Num. obs.	3479	3479	3479	3479	3479	3479	3479	3479
Restrictions:	coefficients a	ssociated with	CS and YS are	e zero.				
LR test $(\chi^2(2))$	1.1983	0.1705	1.0501	1.3037	1.1521	1.1047	0.5172	0.0760
<i>p</i> -value	0.5493	0.9183	0.5915	0.5211	0.5621	0.5756	0.7721	0.9627

Table 8: Factor Models for Country ETF Returns

	US	UK	CA	GER	FRA	ITA	AUS	JAP
(Intercept)	0.0001	0.0000	0.0001	0.0000	-0.0001	-0.0002	0.0002	-0.0001
	(0.0001)	(0.0001)	(0.0002)	(0.0002)	(0.0001)	(0.0002)	(0.0002)	(0.0002)
WOD	2.0914^{***}	2.3373^{***}	1.9570^{***}	2.7898^{***}	2.6177^{***}	2.3375^{***}	2.1476^{***}	2.1149^{***}
	(0.0501)	(0.0807)	(0.0809)	(0.0893)	(0.0645)	(0.0765)	(0.1092)	(0.0888)
VIX	-0.1151^{***}	-0.0845^{***}	-0.0596^{***}	-0.0861^{***}	-0.0979^{***}	-0.1110^{***}	-0.1109^{***}	-0.0601^{***}
	(0.0059)	(0.0083)	(0.0090)	(0.0106)	(0.0088)	(0.0111)	(0.0124)	(0.0104)
DXY	0.6751^{***}	-0.6348^{***}	-0.6780^{***}	-1.2796^{***}	-1.4105^{***}	-1.9697^{***}	-1.1429^{***}	-0.1639
	(0.0569)	(0.0890)	(0.0866)	(0.0927)	(0.0895)	(0.1026)	(0.1069)	(0.1029)
COM	-0.0660^{***}	0.0103	0.4208^{***}	-0.1313^{***}	-0.0878^{***}	-0.0534	0.1835^{***}	-0.1305^{***}
	(0.0212)	(0.0343)	(0.0324)	(0.0348)	(0.0315)	(0.0363)	(0.0426)	(0.0337)
SMB	0.0013^{***}	-0.0007^{**}	0.0016^{***}	0.0006^{*}	0.0003	0.0013^{***}	0.0009^{**}	0.0005
	(0.0003)	(0.0005)	(0.0004)	(0.0006)	(0.0004)	(0.0004)	(0.0006)	(0.0005)
HML	0.0004^{**}	0.0018^{***}	0.0010^{***}	0.0009^{***}	0.0020^{***}	0.0035^{***}	0.0036^{***}	-0.0000
	(0.0003)	(0.0004)	(0.0004)	(0.0003)	(0.0004)	(0.0005)	(0.0005)	(0.0004)
\mathbb{R}^2	0.8447	0.7208	0.6419	0.7584	0.7706	0.6872	0.6145	0.5261
Adj. \mathbb{R}^2	0.8444	0.7204	0.6413	0.7580	0.7702	0.6867	0.6139	0.5253
Num. obs.	3479	3479	3479	3479	3479	3479	3479	3479

Table 9: Factor Models for County ETF Returns with CS and YS Excluded

				No. of	Regimes			
	1	2	3	4	5	6	7	8
Six Factors - (COM + SMB + HML)	-183001.00	-190157.60	-191368.10	-191939.50	-192639.60	-193108.00	-193426.60	-193444.70
Six Factors - $(SMB + HML)$	-183323.10	-190549.40	-191723.60	-192247.80	-192830.40	-193256.00	-193573.40	-193297.00
Six Factors - $(COM + HML)$	-183060.50	-190314.60	-191485.40	-192031.30	-192564.80	-193055.90	-193299.60	-192763.40
Six Factors - (COM + SMB)	-183378.40	-190426.50	-191594.30	-192218.40	-192622.40	-193146.50	-193571.10	-193335.50
Six Factors - HML	-183385.40	-190709.40	-191838.90	-192381.30	-192838.50	-193280.10	-193472.70	-192918.80
Six Factors - SMB	-183686.30	-190762.10	-191875.40	-192447.20	-192943.80	-193354.20	-193463.00	-193448.00
Six Factors - COM	-183460.10	-190600.00	-191755.40	-192333.80	-193352.20	-193133.70	-193232.10	-193210.80
Six Factors	-183770.30	-190936.10	-192023.50	-192872.30	-193516.20	-193620.10	-193584.00	-193449.70
Six Factors + CS	-183706.90	-190816.60	-191839.80	-192754.40	-193306.00	-193348.30	-193236.20	-193054.60
Six Factors + YS	-183709.60	-190824.70	-191852.40	-192645.30	-193231.30	-193299.70	-193204.60	-193005.40
Six Factors + (CS + YS)	-183646.40	-190702.00	-191674.20	-192539.80	-193023.20	-193016.20	-192841.70	-192569.80

Table 10: Bayesian Information Criterion

Notes: The six factor RS model with six regimes is the most preferred model for the country ETF returns as this model has the lowest BIC (-193620.10). The six factors of this preferred model are WOD, VIX, DXY, COM, SMB, and HML. In the preliminary analysis of the single regime factor models for country ETF returns, it is identified that some factors, such as CS and YS, are statistically insignificant for all country ETF returns while some other factors, such as COM, SMB, and HML, are statistically insignificant for some country ETF returns. Using the six factor RS model as the baseline model, other RS models are found by removing, respectively, COM, SMB, HML, COM + SMB, COM + HML, HML + SMB, or COM + HML + SMB from, or by adding, respectively, CS, YS, or CS + YS to the six factor RS model. This table lists the BICs for these models. The BICs support the optimal choice of the six factor RS model of six regimes for the country ETF returns.

-		bull	weakly bull	transition 1	transition 2	weakly bear	bear
	state	1	2	3	4	5	6
bull	1	0.27533	0.59136	0.01012	0.12319	0.00000	0.00000
weakly bull	2	0.42160	0.50292	0.03682	0.03866	0.00000	0.00000
transition 1	3	0.04842	0.08714	0.40629	0.43532	0.00522	0.01761
transition 2	4	0.24522	0.14544	0.44252	0.12552	0.00000	0.04130
weakly bear	5	0.00000	0.00277	0.00000	0.00000	0.96961	0.02761
bear	6	0.00000	0.00000	0.10131	0.02310	0.12166	0.75393

Table 11: Transtion Probabilities of 6-regime Factor Model for Country ETF Returns

Notes: The element in *j*th row and *i*th column represents the transition probability from regime j (j = 1 to 6) to regime i (i = 1 to 6).

Table 12: Mean Changes/Returns of Common Factors across Regimes

-			0 /					0	
	Regime	WOD	VIX	DXY	COM	SMB	HML	\mathbf{CS}	YS
bull	1	0.000312	-0.001704	-0.000051	0.000388	0.038914	0.015393	0.973102	2.635384
weakly bull	2	0.000295	0.000013	-0.000075	0.000261	0.006894	0.035972	0.968341	2.584878
transition 1	3	0.000059	-0.004621	-0.000092	-0.000133	0.062131	0.000625	1.487415	3.036716
transition 2	4	-0.000661	0.007121	0.000133	0.000193	-0.088233	-0.036972	1.308076	3.098212
weakly bear	5	-0.000105	-0.000268	-0.000106	0.000224	0.061567	0.077875	1.036175	2.507160
bear	6	-0.001223	0.003779	0.000169	-0.002023	-0.103072	-0.102952	1.863313	2.983104

Notes: The means of changes/returns of commmon factors across six regimes.

Table 13: Mean Returns of Country ETFs across Regimes

					-		-		
	Regime	US	CA	UK	GER	\mathbf{FRA}	ITA	AUS	JAP
bull	1	0.001889	0.001458	0.002848	0.005249	0.005169	0.005790	0.001484	0.000083
weakly bull	2	0.000187	0.000892	-0.000129	-0.001066	-0.001344	-0.001749	0.001056	0.000913
transition 1	3	0.002877	0.001027	0.004251	0.006399	0.006872	0.007962	0.003089	0.002497
transition 2	4	-0.004546	-0.003574	-0.007427	-0.010550	-0.011158	-0.013577	-0.005280	-0.004021
weakly bear	5	-0.000115	0.000227	-0.000061	-0.000032	-0.000039	0.000144	0.000616	-0.000453
bear	6	-0.003196	-0.003330	-0.003432	-0.006560	-0.004889	-0.004691	-0.003808	-0.002500

Notes: The mean returns on the U.S., Canada, U.K., Germany, France, Italy, Australia, and Japan country ETFs. across six regimes.

Table 14: The Likelihood Ratio Tests for All ETFs across All Regimes

	WOD	VIX	DXY	COM	SML	HML
Likelihood ratio test $(df=48)$	7176.00200	1231.44500	1386.19100	531.67500	311.12400	385.28700
<i>p</i> -value	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000

Notes: The likelihood ratio tests are implemented by comparing the full model with the restricted model where the coefficient of the factor (WOD, VIX, DXY, SML, and HML, respectively) is restricted to be zero for all ETFs. The degrees of freedom for each test is 48. These zero restrictions can be rejected at the 5% significance level.

Table 15: The Likelihood Ratio Tests for Excluding Each Factor for Each ETF across All Regimes

Country							
\mathbf{ETF}	Zero restriction on factor	WOD	VIX	DXY	COM	SMB	HML
US	Likelihood ratio test $(df = 6)$	2673.705	838.662	606.577	280.114	1383.999	265.497
	p-value	0.000	0.000	0.000	0.000	0.000	0.000
CAN	Likelihood ratio test $(df = 6)$	961.530	353.377	383.340	629.047	297.088	63.586
	p-value	0.000	0.000	0.000	0.000	0.000	0.000
$\mathbf{U}\mathbf{K}$	Likelihood ratio test $(df = 6)$	2282.958	202.930	458.350	266.655	246.951	45.307
	p-value	0.000	0.000	0.000	0.000	0.000	0.000
GER	Likelihood ratio test $(df = 6)$	2289.635	166.588	541.674	49.251	283.072	18.260
	p-value	0.000	0.000	0.000	0.000	0.000	0.006
\mathbf{FRA}	Likelihood ratio test $(df = 6)$	2421.199	240.440	621.185	38.718	19.594	417.509
	p-value	0.000	0.000	0.000	0.000	0.003	0.000
ITA	Likelihood ratio test $(df = 6)$	1301.580	194.452	614.952	24.927	277.450	379.994
	p-value	0.000	0.000	0.000	0.000	0.000	0.000
AUS	Likelihood ratio test $(df = 6)$	1290.504	377.810	12.018	48.652	28.291	119.605
	p-value	0.000	0.000	0.062	0.000	0.000	0.000
$_{\rm JAP}$	Likelihood ratio test $(df = 6)$	784.099	35.264	15.101	25.659	41.415	4.750
	p-value	0.000	0.000	0.019	0.000	0.000	0.576

Notes: The likelihood ratio tests are implemented by comparing the full model with the restricted model where the coefficient of the factor (WOD, VIX, DXY, SML, and HML, respectively) is restricted to be zero for each ETF. The degrees of freedom for each test is 6. All but the zero restrictions for the factor HML for Japan ETF and the factor DXY for Australia ETF can be rejected at the 5% significance level.

Table 16: Significance of Coefficients Across Regimes (P < 0.001)

-	C	,			0 (/
	WOD	VIX	DXY	COM	SMB	HML
US	SSSSSS	SSSSSS	SSSNSS	NNNNNN	SSSSSN	NNSSSN
CAN	SSSSSS	SSSSNN	SSNNSN	SSSSNN	SNNNNN	SSNSNN
UK	SSSSSS	SSSSNS	SSSSSN	NNNNNN	NNSSNN	SNSNNN
GER	SSSSSS	SSSSNS	SSSSSN	SSNNNN	SNNNSN	NNNNSN
FRA	SSSSSS	SSSSSS	SSSSSN	NNNNNN	NNNNSN	SSNSSN
ITA	SSSSSS	SSSSNS	SSSSSN	NNNNNN	NNNNSN	SSNNSN
JAP	SSSSSS	NNSNNS	NNNNNN	SNNNNN	SNNNNN	NSNNNN
AUS	SSSSSS	SNSSNS	SSNNSN	NSNNNN	SNNNNN	SSNSNN

Notes: "S" represents that the sensitivity coefficient is statistically significant while "N" represents that the sensitivity coefficient is statistically insignificant. Each cell consists of six letters that represents the statistical significance/insignificance across 6 regimes. For example, the risk factor WOD is statistically significant for all country ETFs across all 6 regimes. Hence we see *SSSSSS*. But the commodity price risk factor COM is statistically insignificant for the France (FRA) and Italy (ITA) country ETFs across all 6 regimes. Hence we see *NNNNN*.

Regime	1	2	3	4	5	6
(Intercept)	0.0012^{***}	-0.0004^{***}	0.0021^{***}	-0.0025^{***}	0.0005^{**}	-0.0005
	(0.0001)	(0.0001)	(0.0003)	(0.0003)	(0.0002)	(0.0012)
WOD	1.5119^{***}	1.8703^{***}	1.5795^{***}	1.8031^{***}	1.9124^{***}	2.2217^{***}
	(0.0443)	(0.0402)	(0.0749)	(0.0767)	(0.0551)	(0.1835)
VIX	-0.1037^{***}	-0.0864^{***}	-0.1908^{***}	-0.0846^{***}	-0.0886^{***}	-0.1775^{***}
	(0.0049)	(0.0044)	(0.0113)	(0.0093)	(0.0103)	(0.0385)
DXY	0.3888^{***}	0.4219^{***}	0.4779^{***}	0.3062^{**}	0.6592^{***}	1.2063^{***}
	(0.0549)	(0.0409)	(0.1271)	(0.1050)	(0.0716)	(0.3219)
COM	-0.0174	-0.0445^{**}	0.0186	-0.1039^{*}	-0.0160	-0.2286^{*}
	(0.0182)	(0.0141)	(0.0406)	(0.0418)	(0.0245)	(0.1149)
SMB	0.0023^{***}	0.0021^{***}	0.0015^{***}	0.0024^{***}	-0.0010^{***}	0.0018
	(0.0002)	(0.0002)	(0.0004)	(0.0004)	(0.0003)	(0.0010)
HML	0.0003	0.0006^{**}	0.0029^{***}	0.0046^{***}	-0.0037^{***}	0.0014
	(0.0003)	(0.0002)	(0.0003)	(0.0004)	(0.0003)	(0.0010)
\mathbb{R}^2	0.8748	0.8983	0.9147	0.9068	0.8954	0.8255
Adj. \mathbb{R}^2	0.8738	0.8977	0.9132	0.9050	0.8945	0.8189
Num. obs.	764	1127	352	317	753	166

Table 17: Regime-Switching Factor Model for US ETF Returns across Regimes

Table 18: Regime-Switching Factor Model for Canada ETF Returns across Regimes

Regime	1	2	3	4	5	6
(Intercept)	0.0006^{**}	0.0000	0.0003	-0.0018^{***}	0.0003	0.0009
	(0.0002)	(0.0002)	(0.0005)	(0.0004)	(0.0004)	(0.0016)
WOD	1.5030^{***}	1.8723^{***}	1.6335^{***}	1.5864^{***}	1.6600^{***}	2.3899^{***}
	(0.0931)	(0.0960)	(0.1439)	(0.1292)	(0.1582)	(0.2531)
VIX	-0.0555^{***}	-0.0352^{***}	-0.1669^{***}	-0.0768^{***}	-0.0274	-0.0199
	(0.0102)	(0.0105)	(0.0217)	(0.0157)	(0.0296)	(0.0532)
DXY	-1.0313^{***}	-0.6799^{***}	-0.4745	-0.5256^{**}	-0.7983^{***}	0.2008
	(0.1155)	(0.0974)	(0.2442)	(0.1767)	(0.2055)	(0.4441)
COM	0.3268^{***}	0.5824^{***}	0.7199^{***}	0.2467^{***}	0.1190	0.5166^{**}
	(0.0382)	(0.0337)	(0.0780)	(0.0704)	(0.0704)	(0.1586)
SMB	0.0033^{***}	0.0010^{*}	-0.0010	0.0020^{**}	0.0023^{**}	0.0021
	(0.0005)	(0.0004)	(0.0007)	(0.0008)	(0.0008)	(0.0014)
HML	0.0038^{***}	0.0018^{***}	0.0018^{**}	0.0042^{***}	-0.0015^{*}	-0.0007
	(0.0006)	(0.0005)	(0.0007)	(0.0007)	(0.0007)	(0.0013)
\mathbb{R}^2	0.6964	0.7012	0.8155	0.7838	0.3359	0.7308
Adj. R ²	0.6940	0.6996	0.8123	0.7796	0.3306	0.7206
Num. obs.	764	1127	352	317	753	166

Notes: The numbers in parentheses are standard errors. ***p < 0.001, **p < 0.01, *p < 0.05

Regime	1	2	3	4	5	6
(Intercept)	0.0021^{***}	-0.0009^{***}	0.0040^{***}	-0.0056^{***}	0.0001	0.0000
	(0.0002)	(0.0001)	(0.0005)	(0.0004)	(0.0004)	(0.0018)
WOD	2.1935^{***}	2.2112^{***}	1.7040^{***}	2.5224^{***}	2.5419^{***}	2.4991^{***}
	(0.0760)	(0.0642)	(0.1209)	(0.1075)	(0.1253)	(0.2805)
VIX	-0.0523^{***}	-0.0547^{***}	-0.1758^{***}	-0.0737^{***}	0.0422	-0.1997^{***}
	(0.0083)	(0.0070)	(0.0183)	(0.0131)	(0.0234)	(0.0589)
DXY	-0.6830^{***}	-0.8706^{***}	-1.2374^{***}	-0.4920^{***}	-0.6599^{***}	0.4225
	(0.0943)	(0.0652)	(0.2052)	(0.1471)	(0.1628)	(0.4921)
COM	0.0046	0.0361	0.2030^{**}	0.1003	-0.0914	-0.1329
	(0.0312)	(0.0226)	(0.0655)	(0.0586)	(0.0558)	(0.1757)
SMB	-0.0001	-0.0005	-0.0023^{***}	-0.0021^{***}	-0.0003	0.0000
	(0.0004)	(0.0003)	(0.0006)	(0.0006)	(0.0007)	(0.0016)
HML	0.0023^{***}	0.0006	0.0019^{***}	0.0012	0.0014^{*}	0.0009
	(0.0005)	(0.0003)	(0.0006)	(0.0006)	(0.0006)	(0.0015)
\mathbb{R}^2	0.7759	0.8228	0.8458	0.8844	0.5324	0.7415
Adj. R ²	0.7741	0.8219	0.8432	0.8822	0.5287	0.7318
Num. obs.	764	1127	352	317	753	166

Table 19: Regime-Switching Factor Model for UK ETF Returns across Regimes

Table 20: Regime-Switching Factor Model for Germany ETF Returns across Regimes

Regime	1	2	3	4	5	6
(Intercept)	0.0045^{***}	-0.0021^{***}	0.0059^{***}	-0.0084^{***}	-0.0002	-0.0037^{*}
	(0.0001)	(0.0001)	(0.0003)	(0.0003)	(0.0004)	(0.0019)
WOD	2.6476^{***}	2.7283^{***}	2.4332^{***}	2.7582^{***}	3.6831^{***}	2.1961^{***}
	(0.0679)	(0.0609)	(0.0901)	(0.0974)	(0.1278)	(0.2955)
VIX	-0.0556^{***}	-0.0578^{***}	-0.0937^{***}	-0.0486^{***}	0.0229	-0.3260^{***}
	(0.0075)	(0.0067)	(0.0136)	(0.0119)	(0.0239)	(0.0621)
DXY	-1.2823^{***}	-1.3607^{***}	-2.4568^{***}	-2.0160^{***}	-0.8969^{***}	-0.3880
	(0.0842)	(0.0618)	(0.1529)	(0.1333)	(0.1660)	(0.5184)
COM	-0.1945^{***}	-0.1067^{***}	0.0869	-0.1281^{*}	-0.1192^{*}	-0.3544
	(0.0278)	(0.0214)	(0.0488)	(0.0531)	(0.0569)	(0.1851)
SMB	0.0012^{***}	0.0005^{*}	0.0006	0.0003	0.0032^{***}	-0.0026
	(0.0004)	(0.0003)	(0.0005)	(0.0006)	(0.0007)	(0.0017)
HML	-0.0000	0.0005	0.0005	0.0007	0.0040^{***}	-0.0006
	(0.0004)	(0.0003)	(0.0004)	(0.0005)	(0.0006)	(0.0015)
\mathbb{R}^2	0.8635	0.8879	0.9314	0.9232	0.6813	0.7425
Adj. R ²	0.8624	0.8873	0.9303	0.9217	0.6787	0.7328
Num. obs.	764	1127	352	317	753	166

Notes: The numbers in parentheses are standard errors. $^{***}p < 0.001, \,^{**}p < 0.01, \,^{*}p < 0.05$

Regime	1	2	3	4	5	6
(Intercept)	0.0045***	-0.0023^{***}	0.0064***	-0.0091^{***}	-0.0003	-0.0019
	(0.0001)	(0.0001)	(0.0003)	(0.0003)	(0.0004)	(0.0017)
WOD	2.6335^{***}	2.4751^{***}	2.3648^{***}	2.5525^{***}	3.5760^{***}	2.0610^{***}
	(0.0589)	(0.0543)	(0.0900)	(0.0794)	(0.1250)	(0.2641)
VIX	-0.0584^{***}	-0.0830^{***}	-0.1552^{***}	-0.0783^{***}	0.0953^{***}	-0.3153^{***}
	(0.0065)	(0.0059)	(0.0136)	(0.0097)	(0.0234)	(0.0555)
DXY	-1.1674^{***}	-1.4063^{***}	-2.1945^{***}	-2.0751^{***}	-0.9664^{***}	-1.1938^{*}
	(0.0731)	(0.0551)	(0.1528)	(0.1086)	(0.1624)	(0.4633)
COM	-0.0686^{**}	-0.0487^{*}	0.0524	-0.0474	-0.0354	-0.3869^{*}
	(0.0242)	(0.0191)	(0.0488)	(0.0433)	(0.0556)	(0.1654)
SMB	-0.0003	-0.0003	-0.0003	-0.0014^{**}	0.0030^{***}	-0.0010
	(0.0003)	(0.0002)	(0.0005)	(0.0005)	(0.0007)	(0.0015)
HML	0.0013^{***}	0.0017^{***}	0.0012^{**}	0.0015^{***}	0.0037^{***}	0.0015
	(0.0004)	(0.0003)	(0.0004)	(0.0004)	(0.0006)	(0.0014)
\mathbb{R}^2	0.8920	0.9065	0.9356	0.9482	0.6470	0.7762
Adj. R ²	0.8912	0.9060	0.9345	0.9472	0.6442	0.7677
Num. obs.	764	1127	352	317	753	166

Table 21: Regime-Switching Factor Model for France ETF Returns across Regimes

Regime	1	2	3	4	5	6
(Intercept)	0.0050***	-0.0029^{***}	0.0074^{***}	-0.0114^{***}	-0.0000	-0.0009
	(0.0002)	(0.0002)	(0.0005)	(0.0005)	(0.0004)	(0.0019)
WOD	2.4452^{***}	2.4140^{***}	2.2128^{***}	2.3597^{***}	2.6730***	2.2589***
	(0.0940)	(0.0891)	(0.1372)	(0.1560)	(0.1358)	(0.3059)
VIX	-0.0692^{***}	-0.0881^{***}	-0.1861^{***}	-0.1056^{***}	0.0826^{**}	-0.2232^{***}
	(0.0103)	(0.0097)	(0.0207)	(0.0190)	(0.0254)	(0.0643)
DXY	-1.4738^{***}	-1.7737^{***}	-3.0405^{***}	-2.9107^{***}	-1.1685^{***}	-1.5750^{**}
	(0.1166)	(0.0905)	(0.2328)	(0.2134)	(0.1764)	(0.5368)
COM	-0.0381	-0.0691^{*}	0.0750	-0.1501	-0.0356	-0.2964
	(0.0385)	(0.0313)	(0.0743)	(0.0850)	(0.0604)	(0.1917)
SMB	-0.0001	-0.0001	-0.0007	-0.0005	0.0026^{***}	0.0025
	(0.0005)	(0.0004)	(0.0007)	(0.0009)	(0.0007)	(0.0017)
HML	0.0033^{***}	0.0036^{***}	0.0020**	0.0024^{**}	0.0031^{***}	0.0020
	(0.0006)	(0.0004)	(0.0006)	(0.0009)	(0.0006)	(0.0016)
\mathbb{R}^2	0.7725	0.8003	0.8783	0.8437	0.4584	0.7095
Adj. R ²	0.7707	0.7992	0.8762	0.8406	0.4540	0.6986
Num. obs.	764	1127	352	317	753	166

Notes: The numbers in parentheses are standard errors. ***p < 0.001, **p < 0.01, *p < 0.05

Regime	1	2	3	4	5	6
(Intercept)	0.0006^{*}	0.0000	0.0020**	-0.0028^{***}	0.0005	-0.0001
	(0.0003)	(0.0002)	(0.0006)	(0.0006)	(0.0004)	(0.0022)
WOD	2.3027^{***}	2.8524^{***}	2.4672^{***}	2.6468^{***}	1.1850^{***}	1.7593^{***}
	(0.1230)	(0.1144)	(0.1600)	(0.1776)	(0.1548)	(0.3525)
VIX	-0.0491^{***}	-0.0043	-0.2533^{***}	-0.0847^{***}	0.0130	-0.4094^{***}
	(0.0135)	(0.0125)	(0.0242)	(0.0216)	(0.0290)	(0.0741)
DXY	-1.1506^{***}	-0.9379^{***}	-0.1581	-0.7743^{**}	-1.0568^{***}	-1.2071
	(0.1526)	(0.1162)	(0.2715)	(0.2430)	(0.2011)	(0.6185)
COM	0.0211	0.2171^{***}	0.2052^{*}	0.1555	0.0191	0.0524
	(0.0504)	(0.0402)	(0.0867)	(0.0968)	(0.0689)	(0.2208)
SMB	0.0025^{***}	0.0004	-0.0019^{*}	-0.0004	0.0003	-0.0007
	(0.0006)	(0.0005)	(0.0008)	(0.0010)	(0.0008)	(0.0020)
HML	0.0026^{***}	0.0022^{***}	0.0009	0.0049^{***}	0.0004	-0.0019
	(0.0008)	(0.0006)	(0.0007)	(0.0010)	(0.0007)	(0.0018)
\mathbb{R}^2	0.6350	0.6854	0.8346	0.7966	0.1628	0.6847
Adj. \mathbb{R}^2	0.6321	0.6837	0.8318	0.7927	0.1560	0.6728
Num. obs.	764	1127	352	317	753	166
NI -+ (TD)				*** < 0.001	** < 0.01 *	< 0.05

Table 23: Regime-Switching Factor Model for Australia ETF Returns across Regimes

Table 24: Regime-Switching Factor Model for Japan ETF Returns across Regimes

Regime	1	2	3	4	5	6
(Intercept)	-0.0009^{**}	0.0003	0.0022***	-0.0022^{***}	-0.0004	-0.0015
	(0.0003)	(0.0002)	(0.0005)	(0.0006)	(0.0004)	(0.0020)
WOD	2.8066^{***}	2.3271^{***}	1.4799^{***}	2.0747^{***}	2.3682^{***}	1.7703^{***}
	(0.1365)	(0.1143)	(0.1340)	(0.1820)	(0.1493)	(0.3171)
VIX	-0.0274	-0.0091	-0.1001^{***}	-0.0463^{*}	0.0011	-0.2329^{***}
	(0.0150)	(0.0125)	(0.0202)	(0.0222)	(0.0279)	(0.0666)
DXY	0.2407	-0.1136	-0.4339	-0.2559	-0.5318^{**}	0.2331
	(0.1693)	(0.1161)	(0.2273)	(0.2489)	(0.1939)	(0.5564)
COM	-0.2476^{***}	-0.0752	-0.1070	-0.1888	0.0258	-0.2017
	(0.0560)	(0.0402)	(0.0726)	(0.0992)	(0.0664)	(0.1987)
SMB	0.0035^{***}	0.0011^{*}	-0.0017^{*}	-0.0006	0.0022^{**}	-0.0033
	(0.0007)	(0.0005)	(0.0007)	(0.0011)	(0.0008)	(0.0018)
HML	0.0015	-0.0024^{***}	0.0006	0.0021^{*}	0.0003	-0.0004
	(0.0009)	(0.0006)	(0.0006)	(0.0010)	(0.0007)	(0.0017)
\mathbb{R}^2	0.5756	0.5104	0.6531	0.6159	0.4484	0.6032
Adj. R ²	0.5722	0.5078	0.6471	0.6084	0.4440	0.5883
Num. obs.	764	1127	352	317	753	166
Notos:Tho n	umbors in par	nthoses are sta	ndard orrors	***n < 0.001	**n < 0.01 *n	< 0.05

Notes: The numbers in parentheses are standard errors. $^{***}p < 0.001$, $^{**}p < 0.01$, $^*p < 0.05$